

**Guided Study Program in System Dynamics**  
System Dynamics in Education Project  
System Dynamics Group  
MIT Sloan School of Management<sup>1</sup>

Assignment #26

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**Reading Assignment:**

Please refer to Road Maps 8: A Guide to Learning System Dynamics (D-4508-1) and read the following paper from Road Maps 8:

- Building a System Dynamics Model Part One: Conceptualization, by Stephanie Albin (D-4597)

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## Exercises:

In Assignment 16, you read *Building a System Dynamics Model Part 1: Conceptualization*. The exercise in the paper asked you to conceptualize two systems: the yellow fever epidemic system and the heroin-crime system. In the first two exercises of this assignment, you will formulate the equations for the two models.

### 1. Formulation Exercise: Yellow Fever Model

#### Step 1: Conceptualization Review

Read again the yellow-fever system description on pages 365-367 of *Study Notes in System Dynamics* by Michael Goodman and review the conceptualization exercise from assignment 16.

#### Step 2: Mosquito Sector

A. The mosquito sector represents the number of mosquitoes in various stages of their lifetime. First, when mosquitoes are hatched, they are potentially dangerous: that is, they can become infectious if they bite a contagious human. Represent this first stage of the mosquito lifetime by a stock of potentially dangerous mosquitoes, and formulate its inflows and outflows.

*Hint 1:* Remember that we are assuming that the total mosquito population is constant and remains in equilibrium throughout the simulation. Hence, the hatch rate of mosquitoes will be constant, and it will equal a certain fraction of the total mosquito population. Because the population remains in equilibrium, the hatch fraction can be determined as the inverse of the average mosquito lifespan. Assume that the total mosquito population at equilibrium is 500,000 mosquitoes.

*Hint 2:* The table on pages 366-367 of *Study Notes in System Dynamics* shows that mosquitoes only feed on the third, sixth, thirteenth, and eighteenth days of their lives. Therefore, a mosquito that has not bitten a contagious person on the third day is safe because it will not feed again until the sixth day. On the sixth day it can bite again, but, because the mosquito incubation process lasts 12 days, a newly incubating mosquito will not live long enough to infect a human. Hence, if a mosquito does not bite a contagious person in the first three days, it can be considered “safe” for the rest of its life, and it should be removed from the stock of potentially dangerous mosquitoes.

*Hint 3:* Mosquitoes that are potentially dangerous enter incubation if they bite a contagious human. Because not all humans bitten by a mosquito are contagious, a mosquito, on each bite, has a certain chance of biting a contagious person. For now, define this chance as a constant fraction: you will redefine it later when the human sector has been formulated. You will also need to calculate how many times a day each mosquito bites: just divide the total number of bites in a mosquito’s life by the average lifespan. Also, it might be convenient to remember that the number of mosquitoes that enter incubation per each bite of a contagious human is 1.

B. The mosquitoes that have entered incubation are now part of the stock of incubating mosquitoes. The inflow to this stock has already been defined, so all that remains to do is to formulate its outflow: a mosquito leaves the incubation period after an average of 12 days and becomes an infectious mosquito.

C. Because it takes three days for a mosquito to feed and become infected, and then 12 days for the virus to develop, the number of days remaining for an infectious mosquito is three days (remember that a mosquito's lifetime is 18 days).

### Step 3: Human Sector

A. Before the yellow fever starts to spread, every person is vulnerable to the disease. The city's population is initially 20,000 people. A person leaves the stock of vulnerable humans and enters the incubation period if he or she is bitten by an infectious mosquito.

*Hint:* In order to define the rate of humans entering incubation, you will need to find the probability that a mosquito bites a vulnerable person. This is just the ratio of vulnerable people to the total human population. For now, leave the human population as a constant: you will later reformulate it as the sum of all the stocks in the chain. You will also need to know how many times per day a mosquito bites a person (you have found this number in Step 2A), and the number of mosquitoes that can transmit the disease (that is, the number of infectious mosquitoes). Finally, it will again help to realize that the number of incubating people resulting from an infectious mosquito's biting one vulnerable person is just 1.

B. Once a person is bitten by an infectious mosquito, he or she stays in the incubation period for 3 to 6 days. Therefore, assume that an incubating person leaves the incubation period after an average of 4.5 days.

*Hint:* Assume that initially, there are 100 incubating people (that is, 100 people who have been bitten and have the yellow fever virus incubating inside them). These people have just come back from a swampy, mosquito-infested area. These 100 people are the root of the epidemic—if there were no incubating people, no one would become contagious, and hence no mosquitoes would become infectious.

C. When the incubation period is over, an incubating person becomes contagious. The period during which a person is contagious lasts 3 to 6 days. Assume again that on average, a person is contagious for 4.5 days. There are initially no contagious people.

D. When the contagion period is over, a contagious person will be sick for 2.5 more days on average (each person is visibly sick for 7 days), but will no longer be able to transmit the disease. A sick person either dies or recovers. Hence, a certain fraction of the sick humans dies after an average of 2.5 days. The rest of the sick people recover over the same time period and become immune for the rest of their lives.

E. Define the total human population as the sum of the five stocks in the chain that you just modeled. Also, define the chance of biting a contagious person as the ratio of contagious people to the total population, and reformulate the rate equation for mosquitoes entering incubation. Finally, redefine the chance of biting a vulnerable person.

In your assignment solutions document, include the model diagram, documented equations, graphs of model behavior, and graphs of lookup functions. Explain the model behavior and relate it to your initial hypothesis.

## 2. Formulation Exercise: Models from Assignment 16

In Assignment 16, you conceptualized two systems, the Russian-pension system and the errant-cats system. In this assignment, choose one of the two systems and formulate the stock and flow structure into a complete model. For the system that you chose, do the following steps:

1. Start by reviewing the stock and flow structure. Does it capture the causal relationships that relate to the problem definition? Recall the intended purpose of the model and make sure that the stock and flow structure is comprehensive enough to achieve that purpose. Revise the original stock and flow structure as needed.
2. Pick a flow that is the easiest to understand. What factors influence the flow? If the flow is a net flow, would it be easier to formulate if you separated it into an inflow and outflow? Does the flow capture one unique, dynamic process or should it be disaggregated into separate flows? Remember that the flow must be in units (of the stock) per time. Add all relevant converters and constants to the model structure and put in the appropriate causal links.
3. Formulate the flow equation. Add auxiliary variables as needed. Keep in mind that equations should give the model unit-consistency. If not, you should analyze whether the inconsistency is due to leaving out vital components, an incomplete understanding of the variable, or some other reason. Return to step 1 and see if any changes need to be made to the overall structure.
4. Repeat steps 2 and 3 for all other flows.
5. The model should now contain equations for all flows and relevant non-constant components. The next step is to find or estimate parameter values to initialize the constants in the system. You may choose to use a number of sources for the information you need, such as books, internet, etc. Enter parameter values into the model. Do the same for initial stock values. Do a units-check in Vensim.
6. If the model contains any nonlinear relationships, formulate the relevant lookup functions. Make sure the lookup functions have dimensionless inputs and outputs.

7. Choose a simulation period that allows you to simulate the system for a long enough time to study the problem being examined. Choose an appropriate time step.
8. Simulate the model and examine the behavior. Explain the behavior. Does the model behavior support your initial hypothesis? Why or why not?
9. Perform sensitivity analysis on the model parameters. What insights have you gained?

In your assignment solutions document, include the model diagram, documented equations, graphs of model behavior, and graphs of lookup functions. Make sure to explain the model behavior and relate it to your initial hypothesis.

*Hints:*

The following includes some of the facts and information sources we used when formulating our models. Some may be helpful to you, some may not be, depending on your specific model purpose and model structure. You may need to approximate or infer many values. Do not feel obligated to include all of the values listed below in your model. Part of the modeling process is filtering through an abundance of data to pick out the relevant data to use. If you need to use parameter values not listed here, do some research on your own. In the documentation of the model, make sure you cite your sources and mention the basis for any assumptions or guesses that you make.

**Excerpt 1: Russia's Pension System**

Some basic information on Russia:  
(obtained from the World Bank Development Report published by the World Bank)

population: 148.2 million (as of 1995)  
population growth fraction: 0.6% (over 1980-1990)  
Annual Per Capita Income: USD 2,240  
PPP per capita: 30.9 where US=100  
life expectancy at birth: 65 years  
total labor force: 77 million (as of 1995)  
growth fraction of labor force: 0.2% (over 1980-1990)

**Excerpt 2: Errant Cats**

Keep in mind that:

- without human intervention the population of cats should be growing rapidly.
- cats' death rate does not depend on prey population, because there does not appear to be an immediate threat of starvation for cats. When one species of prey becomes scarce, cats move onto another species.

You will need to approximate many values in your model. Some of these values will be the result of policy decisions, so you should perform sensitivity analysis on those parameters.

The model behavior does not have to fit historic data exactly. The model should, however, show the general trends in data, for example S-shaped growth, oscillations, or overshoot and collapse.

### 3. Formulation Exercise: Heroin-Crime Model

Read over the description of the heroin-crime system in *Building a System Dynamics Model Part 1: Conceptualization* and review the conceptualization exercise. Then formulate the complete stock-and-flow model for the heroin-crime system into a complete model. In your assignment solutions document, include the model diagram, documented equations, graphs of model behavior, and graphs of lookup functions. Make sure to explain the model behavior and relate it to your initial hypothesis.

For those of you who do not routinely interact with heroin dealers, here are some data relating to drug deals in the Boston area. Although the following data refers to Boston, it can be used for the Detroit modeling:

- Price of heroin: \$150,000 per kilogram
- Amount of heroin necessary to produce a “high” in an average user:  $4 * 10^{-5}$  kilograms per “hit”
- Number of hits a normal heroin addict takes per week: 14
- Average amount of money that an addict makes per revenue-raising crime: \$300

According to our sources of information, the number of heroin addicts in Detroit is approximately 33,000.