

# Guided Study Program in System Dynamics

System Dynamics in Education Project

System Dynamics Group

MIT Sloan School of Management<sup>1</sup>

Solutions to Assignment #26

Tuesday, June 8, 1999

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## ***Reading Assignment:***

*Please refer to Road Maps 8: A Guide to Learning System Dynamics (D-4508-1) and read the following paper from Road Maps 8:*

- *Building a System Dynamics Model Part One: Conceptualization, by Stephanie Albin (D-4597)*

## ***Exercises:***

*In Assignment 16, you read Building a System Dynamics Model Part 1: Conceptualization. The exercise in the paper asked you to conceptualize two systems: the yellow fever epidemic system and the heroin-crime system. In the first two exercises of this assignment, you will formulate the equations for the two models.*

### *1. Formulation Exercise: Yellow Fever Model*

#### *Step 1: Conceptualization Review*

*Read again the yellow-fever system description on pages 365-367 of Study Notes in System Dynamics by Michael Goodman and review the conceptualization exercise from assignment 16.*

#### *Step 2: Mosquito Sector*

*A. The mosquito sector represents the number of mosquitoes in various stages of their lifetime. First, when mosquitoes are hatched, they are potentially dangerous: that is, they can become infectious if they bite a contagious human. Represent this first stage of*

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*the mosquito lifetime by a stock of potentially dangerous mosquitoes, and formulate its inflows and outflows.*

*Hint 1: Remember that we are assuming that the total mosquito population is constant and remains in equilibrium throughout the simulation. Hence, the hatch rate of mosquitoes will be constant, and it will equal a certain fraction of the total mosquito population. Because the population remains in equilibrium, the hatch fraction can be determined as the inverse of the average mosquito lifespan. Assume that the total mosquito population at equilibrium is 500,000 mosquitoes.*

*Hint 2: The table on pages 366-367 of Study Notes in System Dynamics shows that mosquitoes only feed on the third, sixth, thirteenth, and eighteenth days of their lives. Therefore, a mosquito that has not bitten a contagious person on the third day is safe because it will not feed again until the sixth day. On the sixth day it can bite again, but, because the mosquito incubation process lasts 12 days, a newly incubating mosquito will not live long enough to infect a human. Hence, if a mosquito does not bite a contagious person in the first three days, it can be considered “safe” for the rest of its life, and it should be removed from the stock of potentially dangerous mosquitoes.*

*Hint 3: Mosquitoes that are potentially dangerous enter incubation if they bite a contagious human. Because not all humans bitten by a mosquito are contagious, a mosquito, on each bite, has a certain chance of biting a contagious person. For now, define this chance as a constant fraction: you will redefine it later when the human sector has been formulated. You will also need to calculate how many times a day each mosquito bites: just divide the total number of bites in a mosquito’s life by the average lifespan. Also, it might be convenient to remember that the number of mosquitoes that enter incubation per each bite of a contagious human is 1.*

*B. The mosquitoes that have entered incubation are now part of the stock of incubating mosquitoes. The inflow to this stock has already been defined, so all that remains to do is to formulate its outflow: a mosquito leaves the incubation period after an average of 12 days and becomes an infectious mosquito.*

*C. Because it takes three days for a mosquito to feed and become infected, and then 12 days for the virus to develop, the number of days remaining for an infectious mosquito is three days (remember that a mosquito’s lifetime is 18 days).*

### *Step 3: Human Sector*

*A. Before the yellow fever starts to spread, every person is vulnerable to the disease. The city’s population is initially 20,000 people. A person leaves the stock of vulnerable humans and enters the incubation period if he or she is bitten by an infectious mosquito.*

*Hint: In order to define the rate of humans entering incubation, you will need to find the probability that a mosquito bites a vulnerable person. This is just the ratio of vulnerable people to the total human population. For now, leave the human population as a constant: you will later reformulate it as the sum of all the stocks in the chain. You will also need to know how many times per day a mosquito bites a person (you have found this number in Step 2A), and the number of mosquitoes that can transmit the*

disease (that is, the number of infectious mosquitoes). Finally, it will again help to realize that the number of incubating people resulting from an infectious mosquito's biting one vulnerable person is just 1.

B. Once a person is bitten by an infectious mosquito, he or she stays in the incubation period for 3 to 6 days. Therefore, assume that an incubating person leaves the incubation period after an average of 4.5 days.

Hint: Assume that initially, there are 100 incubating people (that is, 100 people who have been bitten and have the yellow fever virus incubating inside them). These people have just come back from a swampy, mosquito-infested area. These 100 people are the root of the epidemic—if there were no incubating people, no one would become contagious, and hence no mosquitoes would become infectious.

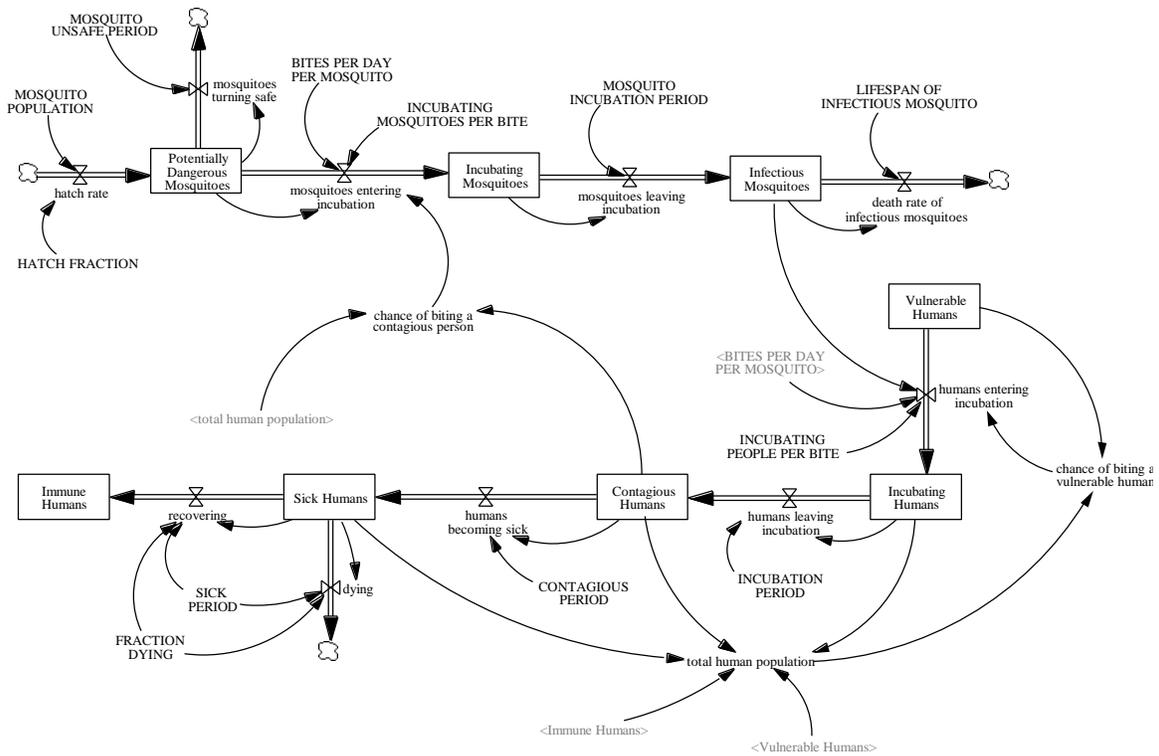
C. When the incubation period is over, an incubating person becomes contagious. The period during which a person is contagious lasts 3 to 6 days. Assume again that on average, a person is contagious for 4.5 days. There are initially no contagious people.

D. When the contagion period is over, a contagious person will be sick for 2.5 more days on average (each person is visibly sick for 7 days), but will no longer be able to transmit the disease. A sick person either dies or recovers. Hence, a certain fraction of the sick humans dies after an average of 2.5 days. The rest of the sick people recover over the same time period and become immune for the rest of their lives.

E. Define the total human population as the sum of the five stocks in the chain that you just modeled. Also, define the chance of biting a contagious person as the ratio of contagious people to the total population, and reformulate the rate equation for mosquitoes entering incubation. Finally, redefine the chance of biting a vulnerable person.

In your assignment solutions document, include the model diagram, documented equations, graphs of model behavior, and graphs of lookup functions. Explain the model behavior and relate it to your initial hypothesis.

Model diagram:



Model equations:

**BITES PER DAY PER MOSQUITO = 0.2222**

Units: (bite/Day)/mosquito

The number of times a mosquito bites a person per day. This value was derived by dividing the total number of bites in a mosquito's life by the average mosquito lifespan (4/18).

**chance of biting a contagious person = Contagious Humans / total human population**

Units: dmnl

The probability that a mosquito bites a contagious person is equal to the ratio of contagious humans to the total human population.

**chance of biting a vulnerable human = Vulnerable Humans/total human population**

Units: dmnl

The probability that a mosquito bites a vulnerable human is equal to the ratio of vulnerable humans to the total human population.

**Contagious Humans = INTEG (humans leaving incubation - humans becoming sick, 0)**

Units: person

The number of people who can infect a mosquito.

**CONTAGIOUS PERIOD = 4.5**

Units: Day

The amount of time during which the virus can be transmitted from a human host to a mosquito host.

death rate of infectious mosquitoes = Infectious Mosquitoes / LIFESPAN OF INFECTIOUS MOSQUITO

Units: mosquito/Day

The rate at which infectious mosquitoes die.

dying = Sick Humans \* FRACTION DYING / SICK PERIOD

Units: person/Day

The rate at which sick people die from yellow fever.

FRACTION DYING = 0.1

Units: dmnl

The fraction of sick people who die.

HATCH FRACTION = 0.0556

Units: 1/Day

Because we are assuming that the total mosquito population is constant, the hatch fraction is determined as the inverse of the average mosquito lifespan (1/18).

hatch rate = MOSQUITO POPULATION \* HATCH FRACTION

Units: mosquito/Day

The number of mosquitoes that hatch per day equals a constant fraction of the total population of mosquitoes.

humans becoming sick = Contagious Humans / CONTAGIOUS PERIOD

Units: person/Day

The rate at which people leave the contagious stage and become sick.

humans entering incubation = Infectious Mosquitoes \* BITES PER DAY PER MOSQUITO \* INCUBATING PEOPLE PER BITE \* chance of biting a vulnerable human

Units: person/Day

The rate at which vulnerable humans become incubating humans. The process of becoming an incubating human begins when a vulnerable person is bitten by an infectious mosquito.

humans leaving incubation = Incubating Humans / INCUBATION PERIOD

Units: person/Day

The rate at which people leave the incubation stage and become contagious. The virus must first incubate in the human host before the human host is capable of spreading the virus.

Immune Humans = INTEG (recovering, 0)

Units: person

The number of people immune to the yellow fever virus. All people who recover from the virus are immune for life.

Incubating Humans = INTEG (humans entering incubation - humans leaving incubation, 100)

Units: person

The number of people who have been bitten and have the yellow-fever virus inside them. Initially, there are 100 incubating people who have just come back from a swampy, mosquito-infested area. These 100 people are the root of the epidemic—if there were no incubating people, no one would become contagious, and hence no mosquitoes would become infectious.

Incubating Mosquitoes = INTEG (mosquitoes entering incubation - mosquitoes leaving incubation, 0)

Units: mosquito

The number of mosquitoes that are in the incubation period.

INCUBATING MOSQUITOES PER BITE = 1

Units: mosquito/bite

The number of mosquitoes that enter incubation (catch the virus) per each bite of a contagious human.

INCUBATING PEOPLE PER BITE = 1

Units: person/bite

The number of incubating people resulting from an infectious mosquito's biting one vulnerable person.

INCUBATION PERIOD = 4.5

Units: Day

The amount of time needed by the virus to develop itself in the human host.

Infectious Mosquitoes = INTEG (mosquitoes leaving incubation - death rate of infectious mosquitoes, 0)

Units: mosquito

The number of mosquitoes that can infect a vulnerable human with yellow fever.

LIFESPAN OF INFECTIOUS MOSQUITO = 3

Units: Day

It takes three days for a mosquito to feed and become infected, and 12 days for the virus to develop. The mosquito's lifetime is 18 days, and, therefore, the number of days remaining for an infectious mosquito is three days.

MOSQUITO INCUBATION PERIOD = 12

Units: Day

The amount of time it takes the virus to develop in a mosquito. After 12 days, the virus is fully developed, and the mosquito can infect a human.

MOSQUITO POPULATION = 500000

Units: mosquito

The total mosquito population is constant at 500,000 mosquitoes.

MOSQUITO UNSAFE PERIOD = 3

Units: Day

Mosquitoes only feed on the third, sixth, thirteenth, and eighteenth days of their lives. Therefore, a mosquito that has not bitten a contagious person on the third day is safe because it will not feed again until the sixth day. On the sixth day it can bite again, but, because the mosquito incubation process lasts 12 days, a newly incubating mosquito will not live long enough to infect a human. Hence, if a mosquito does not bite a contagious person in the first three days, it can be considered safe for the rest of its life.

mosquitoes entering incubation = Potentially Dangerous Mosquitoes \* BITES PER DAY PER MOSQUITO \* INCUBATING MOSQUITOES PER BITE \* chance of biting a contagious person

Units: mosquito/Day

The rate at which mosquitoes enter incubation.

mosquitoes leaving incubation = Incubating Mosquitoes / MOSQUITO INCUBATION PERIOD

Units: mosquito/Day

The rate at which mosquitoes leave the incubation stage to become infectious.

mosquitoes turning safe = Potentially Dangerous Mosquitoes / MOSQUITO UNSAFE PERIOD

Units: mosquito/Day

The rate at which mosquitoes move out of the pool of potentially dangerous mosquitoes. A mosquito can transmit the disease, on average, only if it bites a contagious person during its first blood feed. If a mosquito does not become contaminated during its first blood feed, then it is considered safe.

Potentially Dangerous Mosquitoes = INTEG (hatch rate - mosquitoes entering incubation - mosquitoes turning safe, MOSQUITO POPULATION/6)

Units: mosquito

The number of mosquitoes that can possibly be infected by the yellow-fever virus. The initial value is one-sixth of the total population of mosquitoes because a mosquito is potentially dangerous only during the first three days of its 18-day life.

recovering = Sick Humans \* (1 - FRACTION DYING) / SICK PERIOD

Units: person/Day

The rate at which people recover from yellow fever.

Sick Humans = INTEG (humans becoming sick - dying - recovering, 0)

Units: person

The number of people who exhibit symptoms of yellow fever but are no longer contagious.

SICK PERIOD = 2.5

Units: Day

The amount of time a person is sick but not contagious before dying or recovering.

total human population = Vulnerable Humans + Incubating Humans + Contagious Humans + Sick Humans + Immune Humans

Units: person

The total human population.

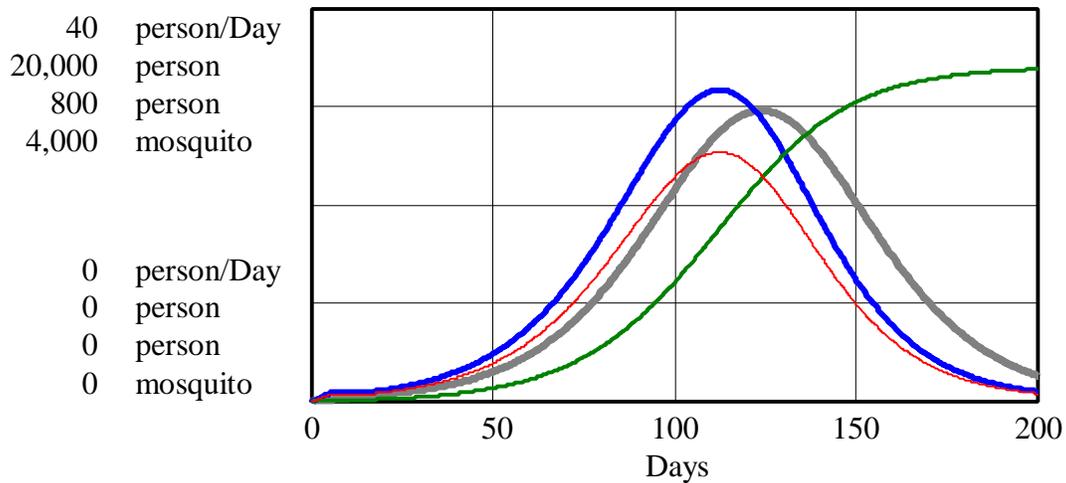
Vulnerable Humans = INTEG (-humans entering incubation, 20000)

Units: person

The number of people who have not been bitten by an infectious mosquito.

Model behavior:

Behavior of yellow-fever model

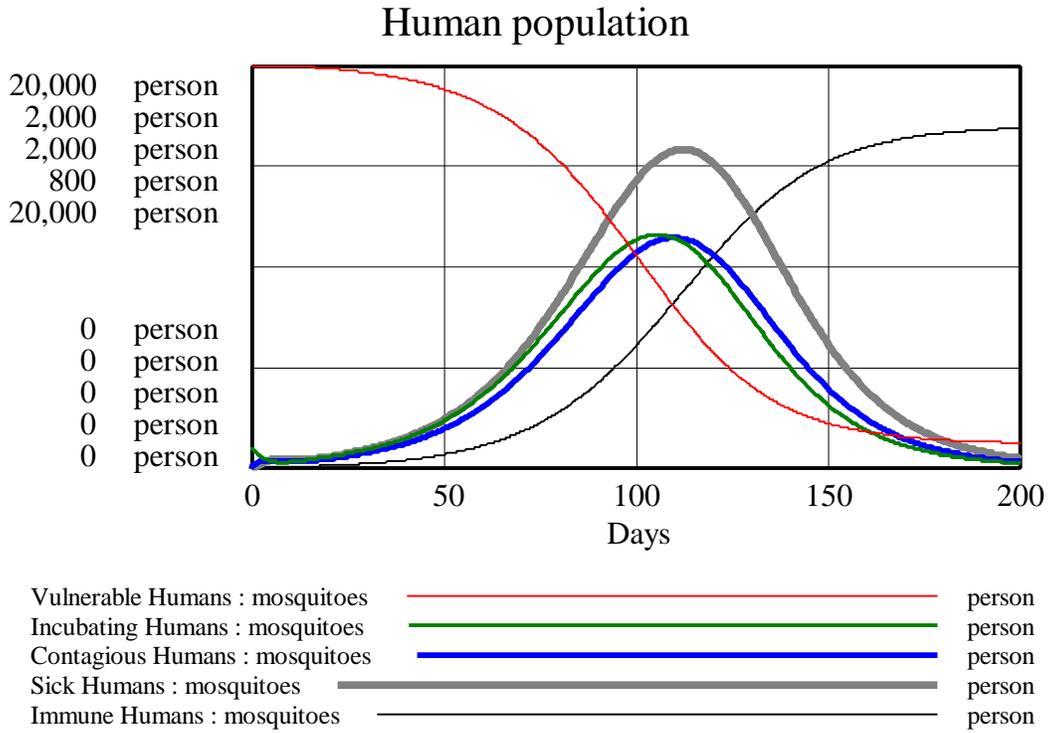


dying : mosquitoes ————— person/Day  
 Immune Humans : mosquitoes ————— person  
 Sick Humans : mosquitoes ————— person  
 Infectious Mosquitoes : mosquitoes ————— mosquito

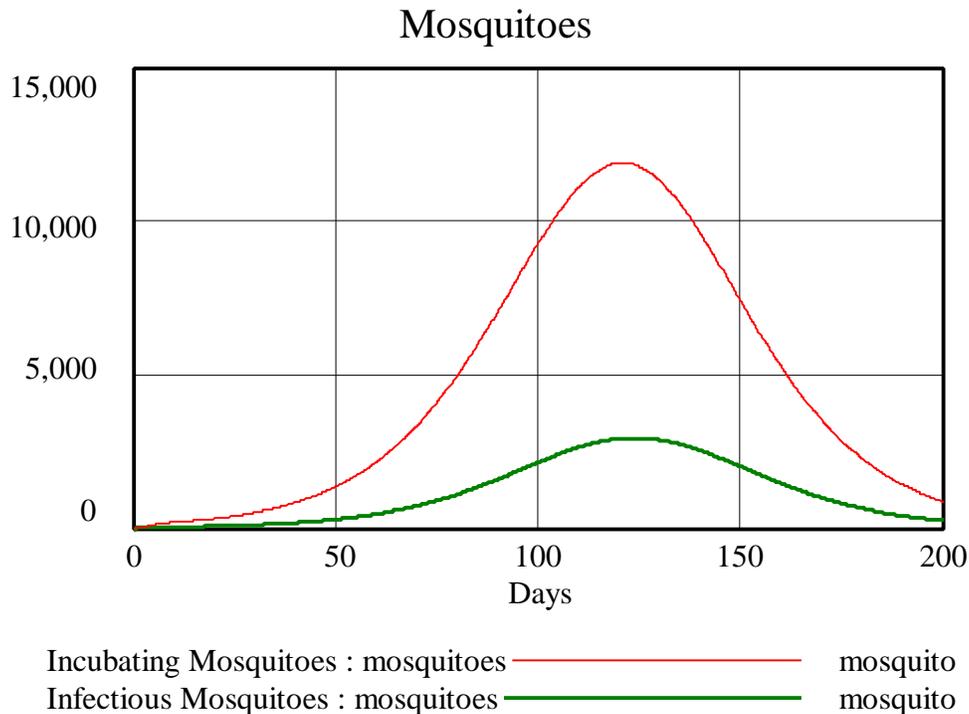
The simulation showed a behavior similar to the reference modes shown in *Building a System Dynamics Model Part 1: Conceptualization*. The system demonstrates the typical S-shaped growth behavior characteristic of an epidemic process.

Initially, the system exhibits exponential growth driven by the positive feedback loop that links “Incubating Humans” to “Contagious Humans” to “Incubating Mosquitoes” to “Infectious Mosquitoes” and back to “Incubating Humans.” An increase in the number of “Incubating Humans” increases the rate of “humans leaving incubation,” so the number of “Contagious Humans” increases. Hence, the “chance of biting a contagious person” is higher, so there are more “mosquitoes entering incubation,” and the number of “Incubating Mosquitoes” increases. There are then more “mosquitoes leaving incubation,” so the number of “Infectious Mosquitoes” rises also, and so does the rate of “humans entering incubation,” which results in a further increase of “Incubating Humans.”

The spread of the virus slows down, however, through a negative feedback loop. As the rate of “humans entering incubation” increases, fewer people remain vulnerable to yellow fever. A smaller number of “Vulnerable Humans” reduces the “chance of biting a vulnerable person,” so there are fewer “humans entering incubation.” The combined effects of these two loops generate an S-shaped growth of the number of “Incubating People,” until “Incubating People” reaches a maximum when the rate of “humans entering incubation” equals the rate of “humans leaving incubation.” When the rate of “humans leaving incubation” exceeds the rate of “humans entering incubation,” the number of “Incubating Humans” starts falling and declines in an S-shaped pattern. The behavior of “Incubating Humans” is therefore a bell-shaped curve. After some time delays, a similar pattern of behavior is transmitted to the other variables in the chain of human population, with the exception of “Immune Humans,” which exhibits S-shaped growth throughout the epidemic. The figure below shows the behavior of the five stocks in the human chain. The three stocks that exhibit bell-shaped behavior (“Incubating Humans,” “Contagious Humans,” and “Sick Humans”) are graphed on the same scale. The S-shaped decline of “Vulnerable Humans” and the S-shaped growth of “Immune Humans” are the basic behaviors generated by many simplified models of an epidemic process.



The stock of “Incubating Mosquitoes” is changed through the rate of “mosquitoes entering incubation,” which depends on the “chance of biting a contagious person.” Because the number of “Contagious Humans” exhibits bell-shaped behavior, so does the “chance of biting a contagious person.” Consequently, the number of “Incubating Mosquitoes” demonstrates the same pattern of behavior, which is then transmitted to the stock of “Infectious Mosquitoes,” as shown in the figure below.



## 2. Formulation Exercise: Models from Assignment 16

*In Assignment 16, you conceptualized two systems, the Russian-pension system and the errant-cats system. In this assignment, choose one of the two systems and formulate the stock and flow structure into a complete model. For the system that you chose, do the following steps:*

- 1. Start by reviewing the stock and flow structure. Does it capture the causal relationships that relate to the problem definition? Recall the intended purpose of the model and make sure that the stock and flow structure is comprehensive enough to achieve that purpose. Revise the original stock and flow structure as needed.*
- 2. Pick a flow that is the easiest to understand. What factors influence the flow? If the flow is a net flow, would it be easier to formulate if you separated it into an inflow and outflow? Does the flow capture one unique, dynamic process or should it be disaggregated into separate flows? Remember that the flow must be in units (of the stock) per time. Add all relevant converters and constants to the model structure and put in the appropriate causal links.*
- 3. Formulate the flow equation. Add auxiliary variables as needed. Keep in mind that equations should give the model unit-consistency. If not, you should analyze whether the inconsistency is due to leaving out vital components, an incomplete understanding of the*

variable, or some other reason. Return to step 1 and see if any changes need to be made to the overall structure.

4. Repeat steps 2 and 3 for all other flows.

5. The model should now contain equations for all flows and relevant non-constant components. The next step is to find or estimate parameter values to initialize the constants in the system. You may choose to use a number of sources for the information you need, such as books, internet, etc. Enter parameter values into the model. Do the same for initial stock values. Do a units-check in Vensim.

6. If the model contains any nonlinear relationships, formulate the relevant lookup functions. Make sure the lookup functions have dimensionless inputs and outputs.

7. Choose a simulation period that allows you to simulate the system for a long enough time to study the problem being examined. Choose an appropriate time step.

8. Simulate the model and examine the behavior. Explain the behavior. Does the model behavior support your initial hypothesis? Why or why not?

9. Perform sensitivity analysis on the model parameters. What insights have you gained?

In your assignment solutions document, include the model diagram, documented equations, graphs of model behavior, and graphs of lookup functions. Make sure to explain the model behavior and relate it to your initial hypothesis.

#### Hints:

The following includes some of the facts and information sources we used when formulating our models. Some may be helpful to you, some may not be, depending on your specific model purpose and model structure. You may need to approximate or infer many values. Do not feel obligated to include all of the values listed below in your model. Part of the modeling process is filtering through an abundance of data to pick out the relevant data to use. If you need to use parameter values not listed here, do some research on your own. In the documentation of the model, make sure you cite your sources and mention the basis for any assumptions or guesses that you make.

#### **Excerpt 1: Russia's Pension System**

Some basic information on Russia:

(obtained from the World Bank Development Report published by the World Bank)

population: 148.2 million (as of 1995)

population growth fraction: 0.6% (over 1980-1990)

Annual Per Capita Income: USD 2,240

PPP per capita: 30.9 where US=100  
 life expectancy at birth: 65 years  
 total labor force: 77 million (as of 1995)  
 growth fraction of labor force: 0.2% (over 1980-1990)

**Excerpt 2: Errant Cats**

Keep in mind that:

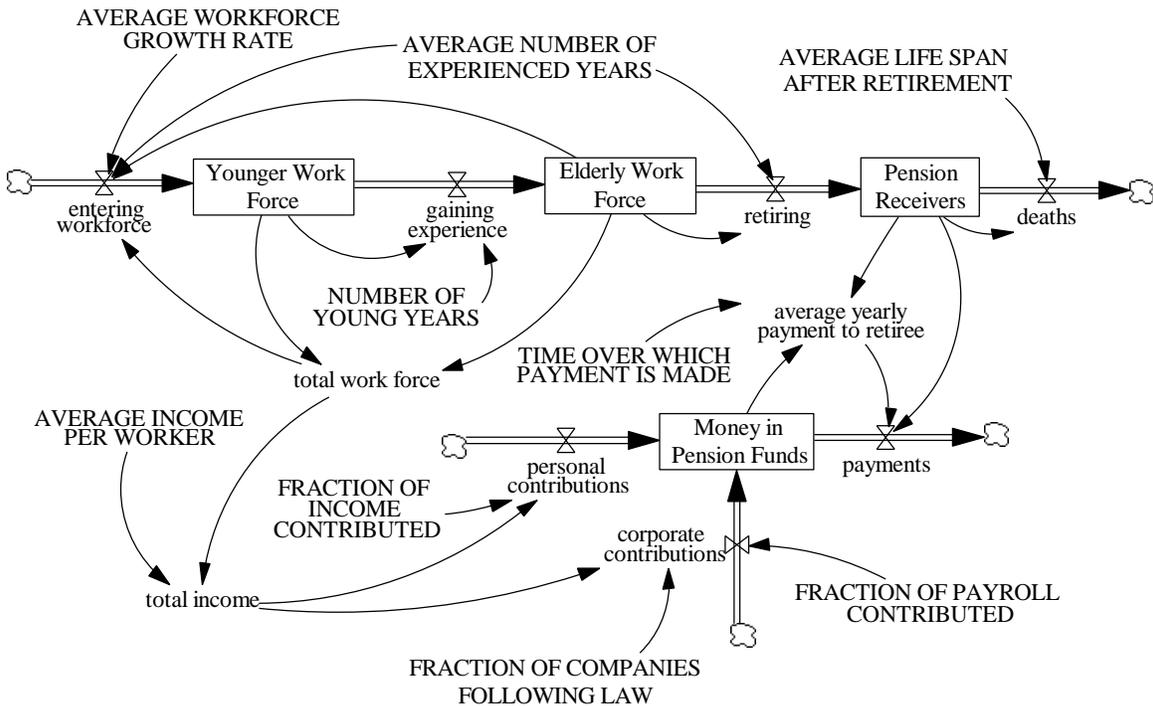
- without human intervention the population of cats should be growing rapidly.
- cats' death rate does not depend on prey population, because there does not appear to be an immediate threat of starvation for cats. When one species of prey becomes scarce, cats move onto another species.

You will need to approximate many values in your model. Some of these values will be the result of policy decisions, so you should perform sensitivity analysis on those parameters.

The model behavior does not have to fit historic data exactly. The model should, however, show the general trends in data, for example S-shaped growth, oscillations, or overshoot and collapse.

Excerpt 1: Russia's Pension System

Model diagram:



Model equations:

AVERAGE INCOME PER WORKER = 4300

Units: dollar / (person \* year)

Average income per worker.

AVERAGE LIFE SPAN AFTER RETIREMENT = 15

Units: year

The number of years a person lives after retirement.

AVERAGE NUMBER OF EXPERIENCED YEARS = 10

Units: year

The average number of years a person is in the experienced work force.

AVERAGE WORKFORCE GROWTH RATE = 0.002

Units: 1/year

The average growth rate of the work force over 1980-90. From the given information, the growth rate is 0.2%.

average yearly payment to retiree = Money in Pension Funds / ((Pension Receivers) \*  
TIME OVER WHICH PAYMENT IS MADE)

Units: dollar / (year \* person)

Pension received by each retiree each year.

corporate contributions = total income \* FRACTION OF PAYROLL CONTRIBUTED \*  
FRACTION OF COMPANIES FOLLOWING LAW

Units: dollar/year

The contributions to the pension fund made by companies each year.

deaths = Pension Receivers / AVERAGE LIFE SPAN AFTER RETIREMENT

Units: person/year

Number of pension-receiving retired workers who die each year.

Elderly Work Force = INTEG (gaining experience - retiring, 2e+007)

Units: person

The total number of workers who are within 10 years of retirement.

entering workforce = (Elderly Work Force / AVERAGE NUMBER OF EXPERIENCED  
YEARS) + (total work force \* AVERAGE WORKFORCE GROWTH RATE)

Units: person/year

The number of young people who take a job each year. It equals the number of people who retire each year plus a fraction of the existing work force.

FRACTION OF COMPANIES FOLLOWING LAW = 0.15

Units: dimensionless

The fraction of companies that follow this particular law regarding pension fund contributions.

FRACTION OF INCOME CONTRIBUTED = 0.01

Units: dimensionless

Fraction of income contributed towards the pension fund by workers.

FRACTION OF PAYROLL CONTRIBUTED = 0.28

Units: dimensionless

The fraction of companies' payroll that the companies should contribute to the pension fund.

gaining experience = Younger Work Force / NUMBER OF YOUNG YEARS

Units: person/year

Number of younger people who become part of more experienced work force each year.

Money in Pension Funds = INTEG (personal contributions – payments + corporate contributions, 2.508e+010)

Units: dollar

The amount of money in Russian pension funds in 1995. Calculated as approximately 1% of the total workforce income in 1995.

NUMBER OF YOUNG YEARS = 25

Units: year

Number of years spent in the younger work force.

payments = average yearly payment to retiree \* Pension Receivers

Units: dollar/year

The total payments made to retired workers out of the pension fund each year.

Pension Receivers = INTEG (retiring - deaths, 3.8e+007)

Units: person

The number of people who receive retirement pensions.

personal contributions = total income \* FRACTION OF INCOME CONTRIBUTED

Units: dollar/year

Total contributions to the pension fund each year made by current workers.

retiring = Elderly Work Force / AVERAGE NUMBER OF EXPERIENCED YEARS

Units: person/year

Number of workers who retire each year.

TIME OVER WHICH PAYMENT IS MADE = 1

Units: year

Time over which the money in the pension fund is paid out to retirees. Under a “pay-as-you-go” system, the entire money in the fund is paid out in the current year.

total income = AVERAGE INCOME PER WORKER \* total work force

Units: dollar/year

Total annual income of the current workforce.

total work force = Younger Work Force + Elderly Work Force

Units: person

Total working population in Russia.

Younger Work Force = INTEG (entering workforce - gaining experience, 5.7e+007)

Units: person

The part of the workforce who are more than 10 years away from the average retirement age.

#### Notes on formulating the model:

While reading through the documentation, it may seem that the equations are contrary to the information provided in the assignment.

1. The data says that annual per capita income = \$2240, while the equations show that AVERAGE INCOME PER WORKER = \$4300 per person per year. This is because:  
average per capita income = total national income / total population

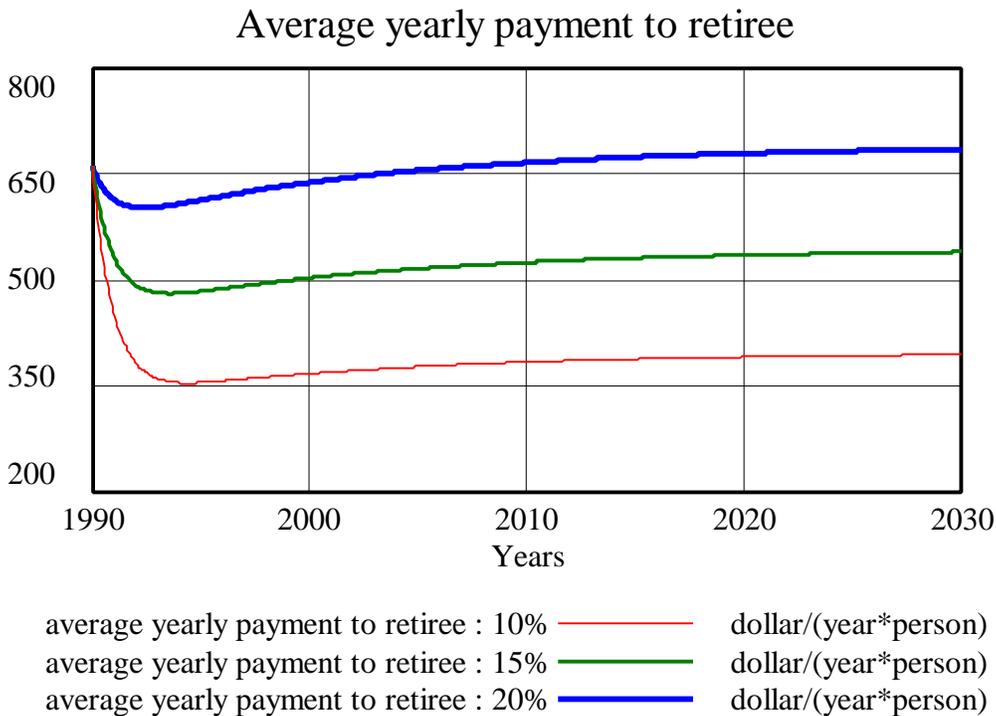
Assuming the total national income is earned only by the working part of the population, we can say that

$$\begin{aligned} \text{AVERAGE INCOME PER WORKER} &= \text{total national income} / \text{total working} \\ &\quad \text{population} \\ &= \text{average per capita income} * \text{total population} / \text{total working population} \\ &= \$4300 \end{aligned}$$

2. The formulation for “entering workforce” may be confusing. The data shows that the workforce is growing at 0.2% per year, that is by 0.002 times the total workforce. This means that the “entering WF” equals the “retiring” flow plus 0.2% of “total work force.”
3. Average retirement age for most people is between 55 and 60 years, and average life expectancy at birth is 65 years. Hence, “AVERAGE LIFE SPAN AFTER RETIREMENT” should be less than 10 years. However, many people retire before they are 55, thereby reducing the average retirement age. Furthermore, 65 years is the average life expectancy at birth. Therefore, the average for people who retire should be higher in order to compensate for those who die at young ages. Due to the absence of hard data, sensitivity tests on this constant could be useful.

4. Initial values of the stocks of workers. The total number of workers equals 77 million. The total number of years an average person works is 35 years. If the demographic distribution is consistent, then the number of workers in each stock of workers will be proportional to the number of years a worker spends in that stock. Hence, in 1995, the value of the stock of Elderly Work Force is  $77M * 10 / 35 = 22M$ . We are also told, however, that due to the large number of men killed during World War II (and the subsequent lower birth rate), the current work force is relatively young. Therefore, we set the “Elderly Work Force” in 1995 equal to 20 Million.

5. The “FRACTION OF COMPANIES FOLLOWING LAW” also has to be estimated. Just because there is no data about this factor does not justify ignoring it! In fact, as the graph below shows, the system behavior is very sensitive to changes in this constant. The graph shows the average yearly payment made to a retiree as the fraction of companies following the law changes even by fairly small percentages.

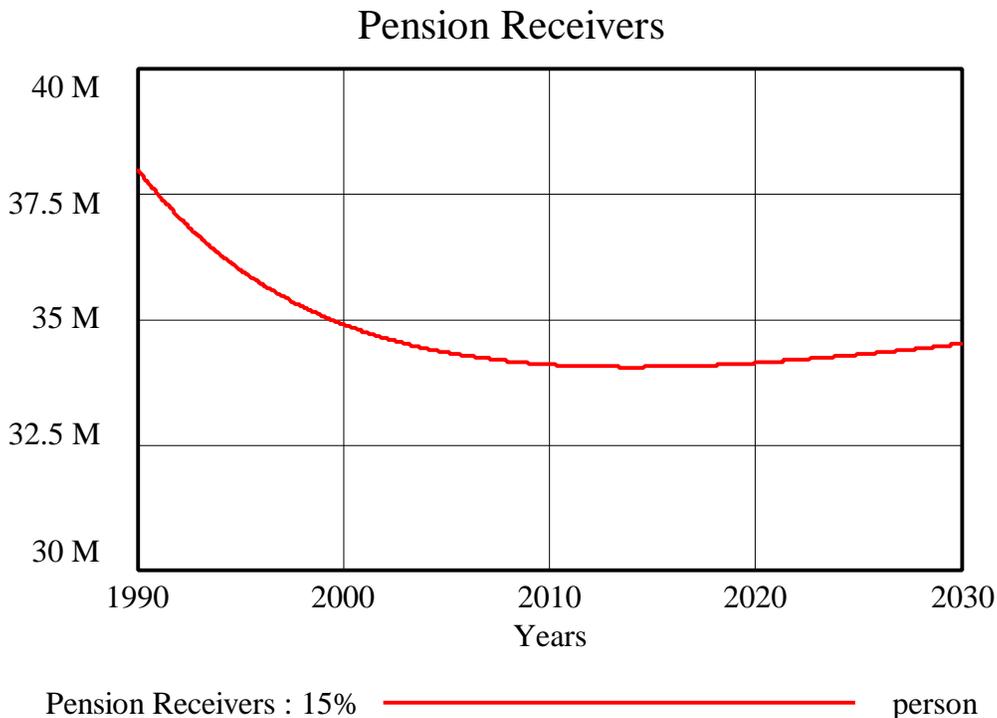


The above graph points to a policy suggestion that could be more effective and easier to implement than a complete overhaul of the system: enforcing the law that companies have to contribute towards the pension fund, and possibly lowering the percentage of payroll that needs to be contributed to encourage companies to follow the law. There are probably many other factors, such as the legal system, possible enforcement methods, and so on that need to be considered before a final conclusion can be reached.

Note here that “FRACTION OF COMPANIES FOLLOWING LAW” may not be the most appropriate name for this constant. It essentially represents the fraction of workers

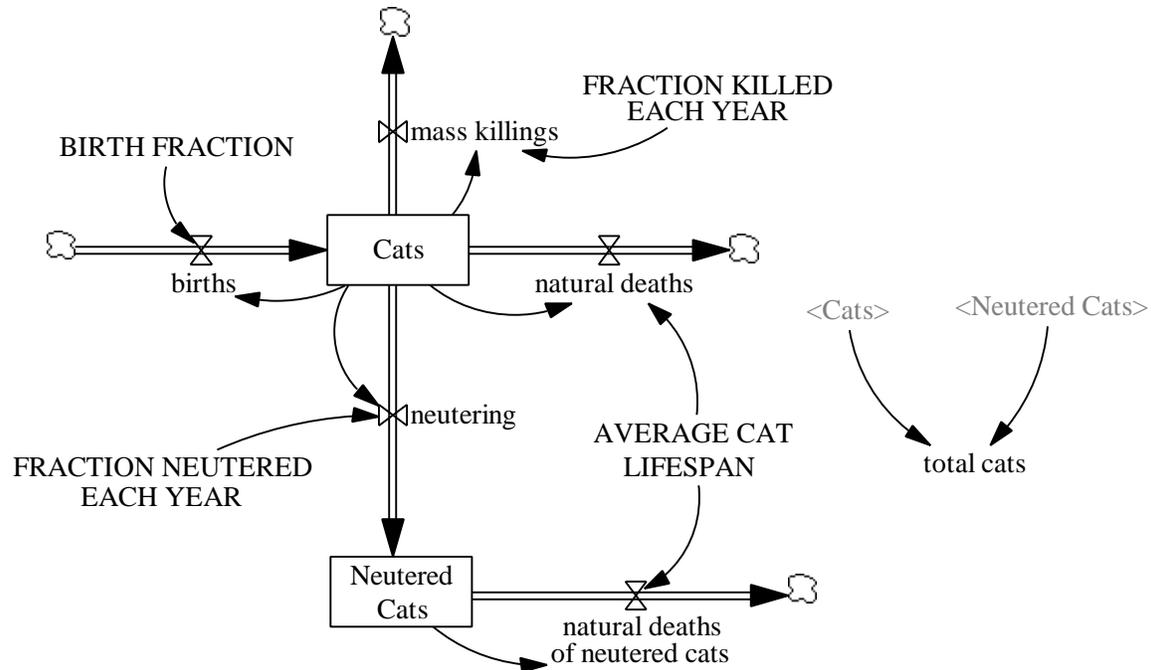
whose companies follow the law. For example, if no large companies (by number of employees) do not follow this law, but all small companies do, then the fraction of companies following the law is large, but 28% of payroll of only a small number of workers is being contributed to the fund. Therefore, the fraction will still be small (This is probably close to the real situation in Russia, where state owned or recently privatized large corporations seem to have the highest levels of disregard for the law).

The following graph shows how the number of pension receivers changes over time. As the system hypothesis suggests, the number of pension receivers decreases from 38 million in 1990 to 34 million in 2014 and then starts to increase again. The years do not exactly coincide with the extract, but the trend in behavior can be seen. Better data could help make the model more accurate. Again, remember that forecasting exact numbers should not always be the goal of building a model.



Excerpt 2: Errant Cats

Model diagram:



Model equations:

AVERAGE CAT LIFESPAN = 10

Units: year  
Average life span of a house cat.

BIRTH FRACTION = 0.5

Units: 1/year  
The cat birth fraction, assuming that two adult cats give birth to four kittens a year, one of which survives to reach the mating age.

births = Cats \* BIRTH FRACTION

Units: cat/year  
Number of cats born each year.

Cats = INTEG (births - mass killings - natural deaths - neutering, 1.2e+007)

Units: cat  
Total number of cats in Australia.

FRACTION KILLED EACH YEAR = 0

Units: 1/year  
Fraction of the cat population killed by people each year.

FRACTION NEUTERED EACH YEAR = 0

Units: 1/year  
Fraction of cats that are neutered each year.

mass killings = Cats \* FRACTION KILLED EACH YEAR

Units: cat/year

Number of cats killed by people each year.

natural deaths = Cats / AVERAGE CAT LIFESPAN

Units: cat/year

Number of cats that die of old age each year.

natural deaths of neutered cats = Neutered Cats / AVERAGE CAT LIFESPAN

Units: cat/year

Number of neutered cats that die of old age each year.

Neutered Cats = INTEG (neutering - natural deaths of neutered cats, 0)

Units: cat

Total number of neutered cats in Australia.

neutering = Cats \* FRACTION NEUTERED EACH YEAR

Units: cat/year

Number of cats that are neutered each year.

total cats = Cats + Neutered Cats

Units: cat

Total number of cats in Australia.

### Simulating the model:

The model is simple to understand. The graphs below show the system response with various policy decisions.

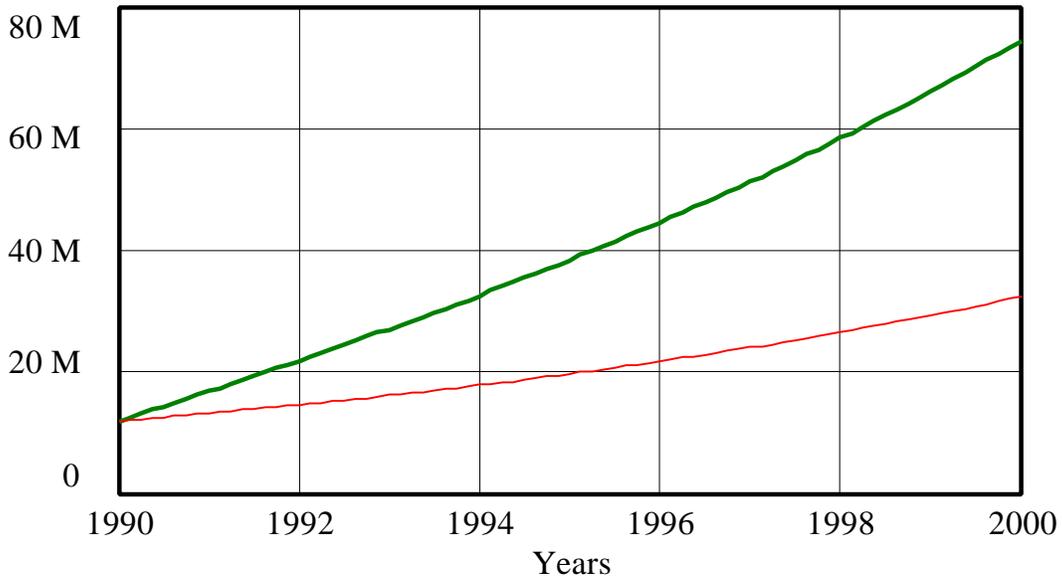
The graph below shows the growth of the cat population if nothing is done to control it. Note that the model does not consider the natural limitation to growth, such as availability of prey, land constraints, and so on.

The following graph shows two policies that can be implemented to control the growth of the cat population:

- killing 30% of the cat population each year (i.e., set FRACTION KILLED EACH YEAR = 0.3 and set FRACTION NEUTERED EACH YEAR = 0)
- neutering 30% of the cat population each year (i.e., set FRACTION NEUTERED EACH YEAR = 0.3 and set FRACTION KILLED EACH YEAR = 0.0).

The policy of both killing and neutering cats is not studied in this document, and is probably the most likely decision. If you decide to try this option, define the process very carefully: are only wild cats killed or will some neutered cats also be killed by angry mobs? If neutered cats are also killed, will the same fraction of neutered cats (who will mostly be pets and hence protected) be killed as wild cats? Depending on answers to these questions, you may have to add extra outflows in the model structure.

### Total Cats - kill or neuter?

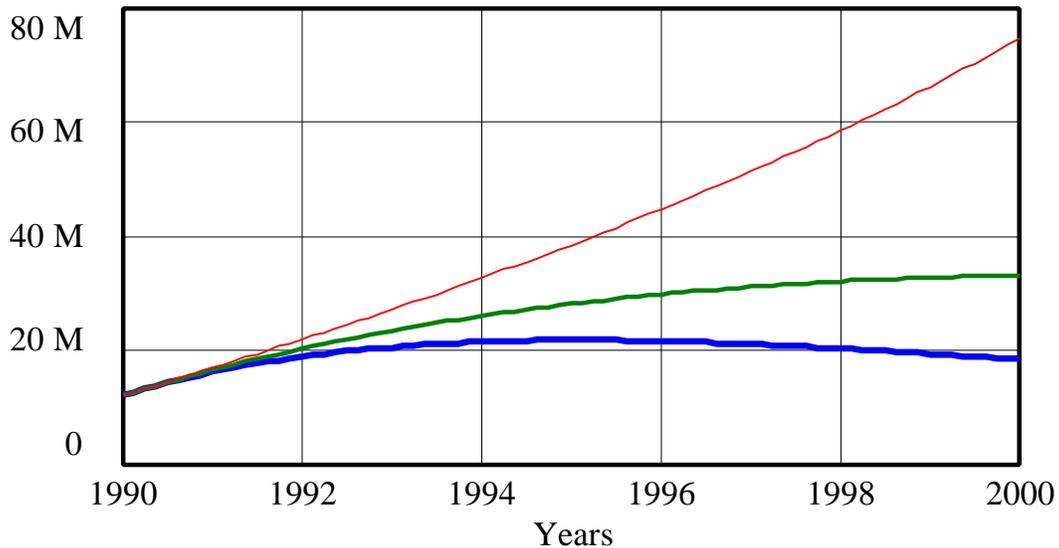


total cats : Kill 30% ———— cat  
total cats : Neuter 30% ———— cat

Clearly, killing large number of cats seems to be a more effective population control method than neutering. Neutering a larger fraction of cats, however, will have the same effect as killing a smaller fraction of cats, and will be the more humane solution.

A high enough fraction of neutering can lead to an asymptotic growth of the cat population to some equilibrium value. The exact equilibrium value depends on the fraction neutered. A very high neutering fraction can lead to the growth and then rapid decline of the total cat population.

## Neutering Cats



total cats : Neuter 30% ———— cat  
 total cats : Neuter 45% ———— cat  
 total cats : Neuter 60% ———— cat

Only killing parts of the cat population can, however, decrease the cat population without the initial increase seen in the run with Neutering fraction = 0.6. Killing cats may be justified in places where they cause real and serious harm, but it should not be used as the primary tool for controlling the population.

### 3. Formulation Exercise: Heroin-Crime Model

*Read over the description of the heroin-crime system in Building a System Dynamics Model Part 1: Conceptualization and review the conceptualization exercise. Then formulate the complete stock-and-flow model for the heroin-crime system into a complete model. In your assignment solutions document, include the model diagram, documented equations, graphs of model behavior, and graphs of lookup functions. Make sure to explain the model behavior and relate it to your initial hypothesis.*

*For those of you who do not routinely interact with heroin dealers, here are some data relating to drug deals in the Boston area. Although the following data refers to Boston, it can be used for the Detroit modeling:*

- *Price of heroin: \$150,000 per kilogram*
- *Amount of heroin necessary to produce a “high” in an average user:  $4 * 10^{-5}$  kilograms per “hit”*

- *Number of hits a normal heroin addict takes per week: 14*
- *Average amount of money that an addict makes per revenue-raising crime: \$300*

*According to our sources of information, the number of heroin addicts in Detroit is approximately 33,000.*

Behavior Hypothesis:

It should be expected that crime increases just after a bust. As the amount of heroin on the market decreases, demand becomes greater than supply, which drives prices up. It should be kept in mind that demand is rather inelastic over short periods of time because of the addictive nature of heroin. Thus, when price rises and people cannot afford to buy heroin, they commit crimes to raise the money to buy the drug, rather than cut down on consumption.

Because drug busts are infrequent events, the supply of heroin should eventually rise to equal demand, and the price of heroin returns to the normal price.

Model diagram:



The number of heroin addicts in Detroit.

Amount of Heroin on Market = INTEG (importation - busts - selling, 73.92)

Units: kilo

The number of kilograms of heroin on the market. Initial value equals the desired heroin amount.

amount of money available for habit = heroin demand \* NORMAL PRICE \*  
FRACTION OF MONEY AVAILABLE NOT FROM CRIME

Units: dollar/week

At the normal price, the addict population needs the amount of money equal to heroin demand times the normal price. It is assumed that the heroin addicts only have a certain fraction of this amount available; the rest of the money must be obtained through crime-related activities. Hence, even in equilibrium with the normal price of heroin, there is some amount of drug-related crime in the city.

AVERAGE AMOUNT OF HEROIN PER HIT = 4e-005

Units: kilo/hit

The amount of heroin required to give an average user a "high."

AVERAGE NUMBER OF HITS PER WEEK = 14

Units: (hit/person)/week

Number of hits a normal addict takes per week.

AVERAGE REVENUE PER CRIME = 300

Units: dollar/crime

The average income from each crime.

busts = Amount of Heroin on Market \* FRACTION OF HEROIN CONFISCATED \*  
PULSE (TIME OF BUST, 1)

Units: kilo/week

The number of kilograms taken from the general supply of heroin available for sale by law enforcement officials. The bust lasts for 1 time period.

change in price = price gap / TIME TO CHANGE PRICE

Units: (dollar/kilo)/week

The rate at which heroin price changes. Changing the price takes time because an increase in demand due to scarcity is not immediately recognized.

desired heroin amount = heroin demand \* SUPPLY COVERAGE

Units: kilo

The amount of heroin the dealers in Detroit would like to have on hand at any time.

desired price = NORMAL PRICE \* effect of heroin amount on price

Units: dollar/kilo

The price a dealer would like to receive for one kilo of heroin.

effect of heroin amount on price = effect of heroin amount on price lookup (ratio of actual to desired heroin amount)

Units: dimensionless

The multiplier showing how the price of heroin depends on the ratio of actual heroin available in the city to the desired heroin amount.

effect of heroin amount on price lookup  $[(0,0) - (2,4)], (0,3.74), (0.2,2.32), (0.4,1.7), (0.6,1.34), (0.8,1.14), (1,1), (1.2,0.82), (1.4,0.68), (1.6,0.54), (1.8,0.36), (2,0.22))$

Units: dimensionless

The lookup function showing how the price of heroin changes depending on the availability of heroin.

effect of heroin amount on sales = effect of heroin amount on sales lookup (ratio of actual to desired heroin amount)

Units: dimensionless

The multiplier showing how sales of heroin depend on the ratio of actual heroin available in the city to the desired heroin amount.

effect of heroin amount on sales lookup  $[(0,0) - (1,1)], (0,0), (0.2,0.22), (0.4,0.615), (0.6,0.88), (0.8,0.97), (1,1), (2,1))$

Units: dimensionless

The lookup function showing how sales change depending on the amount of heroin available.

effect of price on importation = effect of price on importation lookup (ratio of actual price to normal price)

Units: dimensionless

The multiplier showing how the ratio of actual price to normal price affects the amount of heroin imported into Detroit.

effect of price on importation lookup  $[(0,0) - (4,3)], (0,0), (0.5,0.01), (1,1), (1.5,1.5), (2,1.77), (2.5,1.95), (3,2.1), (3.5,2.1), (5,2.1))$

Units: dimensionless

The lookup function showing how the amount of heroin imported into the city depends on price.

FRACTION OF HEROIN CONFISCATED = 0

Units: 1/week

The heroin confiscated per bust per week as a fraction of the total heroin in the market.

FRACTION OF MONEY AVAILABLE NOT FROM CRIME = 0.6

Units: dimensionless

The fraction of money needed to cover the demand at the normal price that the heroin addicts have available without committing crimes.

heroin demand = AVERAGE NUMBER OF HITS PER WEEK \* AVERAGE AMOUNT OF HEROIN PER HIT \* ADDICT POPULATION

Units: kilo/week

The amount of heroin desired by the addict population.

importation = heroin demand \* effect of price on importation

Units: kilo/week

The rate at which heroin is imported into the city depends on the price of heroin. The higher the price, the higher the imports because profits from selling will be higher.

money needed from crime = money needed to support population's habit - amount of money available for habit

Units: dollar/week

The difference between the amount of money required to support the addicts' habits and the amount of money that the addict population earned through non-criminal means.

money needed to support population's habit = heroin demand \* Price

Units: dollar/week

The total amount of money needed to buy enough heroin to satisfy addict demand.

NORMAL PRICE = 150000

Units: dollar/kilo

The price of heroin when supply equals demand.

Price = INTEG (change in price, 150000)

Units: dollar/kilo

The price of one kilogram of heroin. Initial value equals the normal price.

price gap = desired price - Price

Units: dollar/kilo

The difference between the desired and actual price of heroin.

ratio of actual price to normal price = Price / NORMAL PRICE

Units: dimensionless

The ratio of the current price of heroin to normal price.

ratio of actual to desired heroin amount = Amount of Heroin on Market / desired heroin amount

Units: dimensionless

The ratio of current heroin supply to desired heroin supply.

revenue raising crime = money needed from crime/AVERAGE REVENUE PER CRIME

Units: crime/week

The number of crimes committed to get the money required to pay for heroin.

selling = heroin demand \* effect of heroin amount on sales

Units: kilo/week

The rate at which heroin is sold to addicts.

SUPPLY COVERAGE = 4

Units: week

The number of weeks of heroin demand dealers want to keep at hand to deal with sudden changes in demand or supply.

TIME OF BUST = 8

Units: week

The time at which the police conduct a bust.

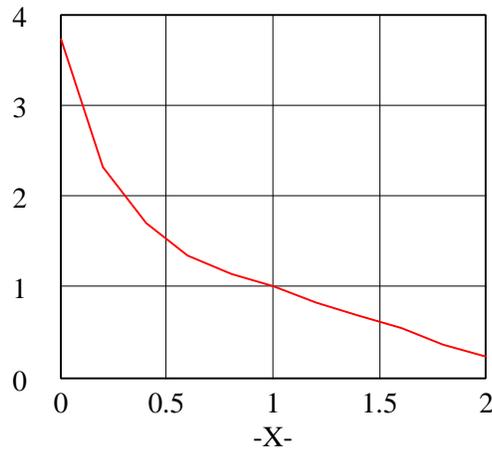
TIME TO CHANGE PRICE = 2

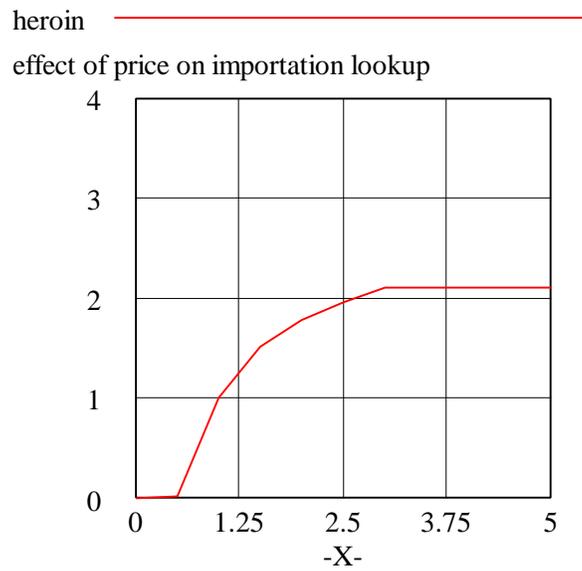
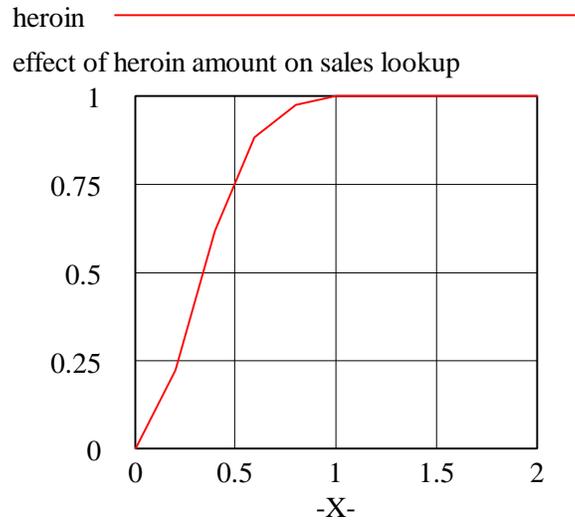
Units: week

The time it takes for dealers to change price in response to imbalances between supply and demand.

heroin —————

effect of heroin amount on price lookup

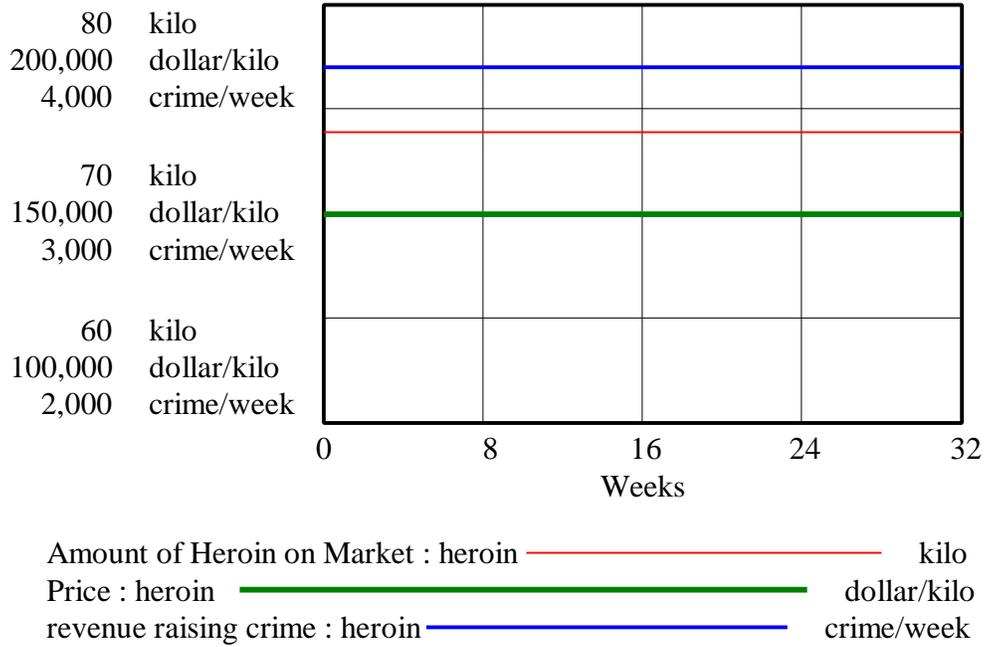




Model Behavior:

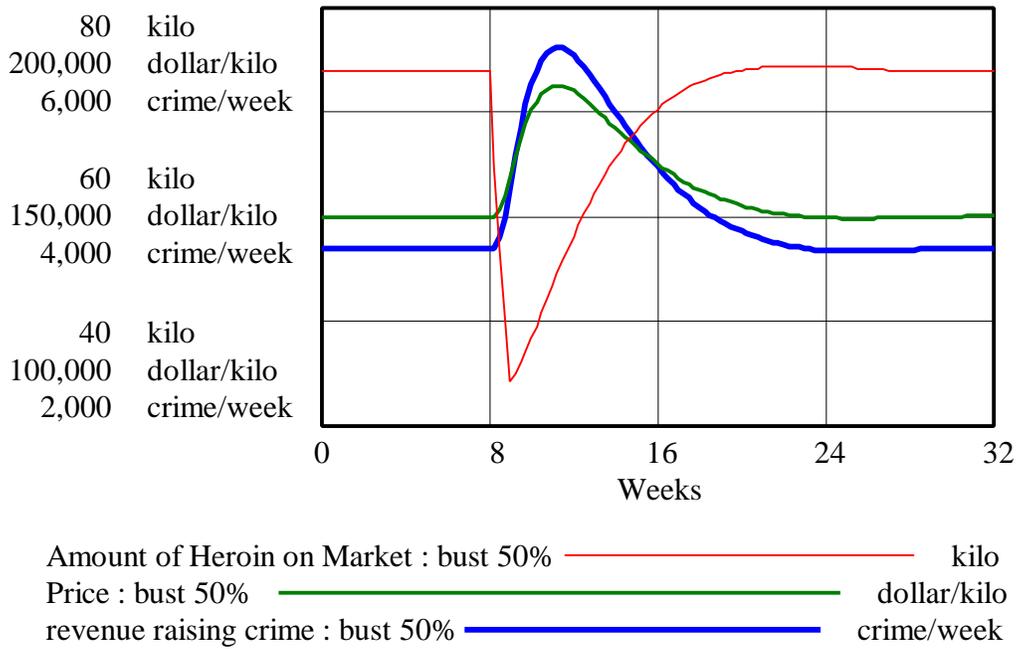
If there are no busts, the system should remain in equilibrium, that is, there should be no change in the system behavior over time.

### Model behavior with no busts



Now, setting “FRACTION OF HEROIN CONFISCATED” to 0.5:

### Drugs seized in bust = 50%



Thus, the system exhibits the behavior that was described in the hypothesis earlier.