Guided Study Program in System Dynamics
System Dynamics in Education Project
System Dynamics Group
MIT Sloan School of Management[1]

Solutions to Assignment #5
October 27, 1998

Reading Assignment:

Please read the following:
- *Principles of Systems,[2] by Jay W. Forrester, Sections 2.1, 2.2, 2.4*

Then refer to Road Maps 3: A Guide to Learning System Dynamics (D-4503-4) and read the following papers from Road Maps 3:

- *Graphical Integration Exercises Part 2: Ramp Functions (D-4571)*
- *Beginner Modeling Exercises Section 2: Mental Simulation of Positive Feedback (D-4487)*
- *Beginner Modeling Exercises Section 3: Mental Simulation of Negative Feedback (D-4536)*

Exercises:

1. Principles of Systems

Please read sections 2.1, 2.2, and 2.4 in Principles of Systems (we will return to sections 2.3 and 2.5 in later assignments) and do the workbook exercises for these sections (located at the end of the book). You should understand all the material in these sections; please let us know if you have any questions. You do not need to submit anything for this reading assignment.

2. Graphical Integration Exercises Part 2: Ramp Functions

A. Using the skills you acquired in Graphical Integration Exercises Part Two, graphically integrate the following flows. Use a graphics application to create the

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[2] Forrester, Jay W., 1968. *Principles of Systems*, (2nd ed.). Portland, Oregon: Productivity Press. 391 pp.

graphs of the stock behaviors, and then paste the graphs into your assignment solutions document.

1. Assume that the initial value of the stock is –50.
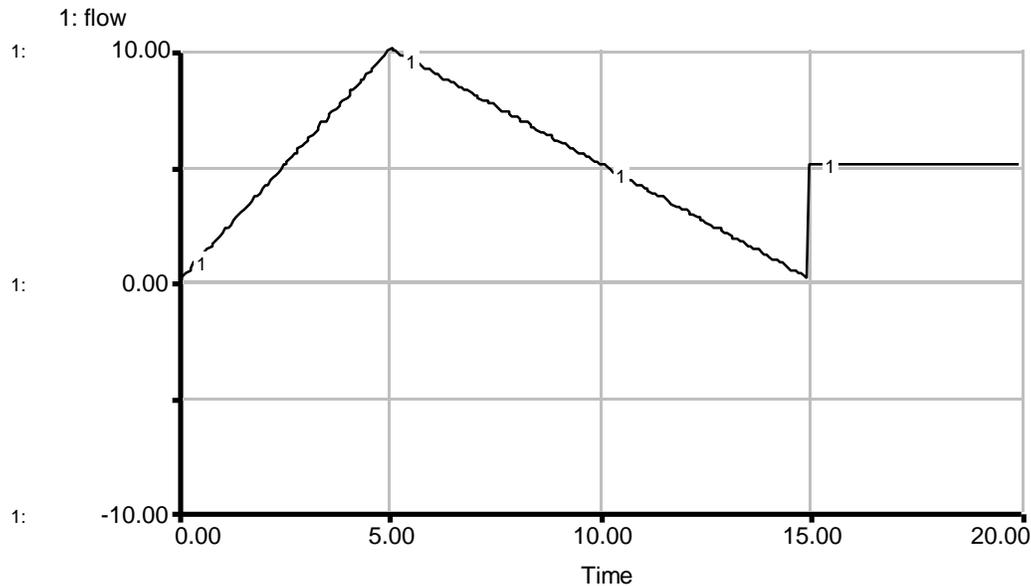
At time =0, the flow is at 0. From time = 0 to time = 5, the flow increases linearly with slope +2. From time = 5 to time = 15, the flow remains positive but decreases linearly with slope –1. The flow then steps up to +5 at time = 15 and remains constant at +5 until time = 20.

The initial value of the stock is –50, and the flow starts at 0.

From time = 0 to time = 5, the flow increases linearly with slope +2. The change in the value of the stock, which equals the area under the graph of the flow, is 5 * 10 / 2 = 25 units of stock. The value of the stock therefore increases by 25 units, to –25. Because the value of the flow increases during this time segment, the slope of the graph of the stock also increases, so the stock exhibits parabolic growth.

From time = 5 to time = 15, the flow remains positive but decreases linearly with slope –1. The change in the value of the stock, which equals the area under the flow graph, is (15 – 5) * 10 / 2 = 50 units of stock. The value of the stock thus increases by 50 units (remember that the flow is positive, so the stock must increase), from –25 to 25 units. Because the value of the flow decreases during this time segment, the slope of the graph of the stock also decreases, so the stock exhibits "decreasing" parabolic growth.

The flow then steps up to 5 at time = 15 and remains constant at 5 until time = 20. The change in the value of the stock, which equals the area under the flow graph, is (20 – 15) * 5 = 25 units of stock. The value of the stock increases by 25 units, from 25 to 50 units.

Because the flow is constant (its value does not change) during this time segment, the slope of the graph of the stock also remains constant, so the stock grows linearly.

The stock behavior corresponding to the above description is shown below:
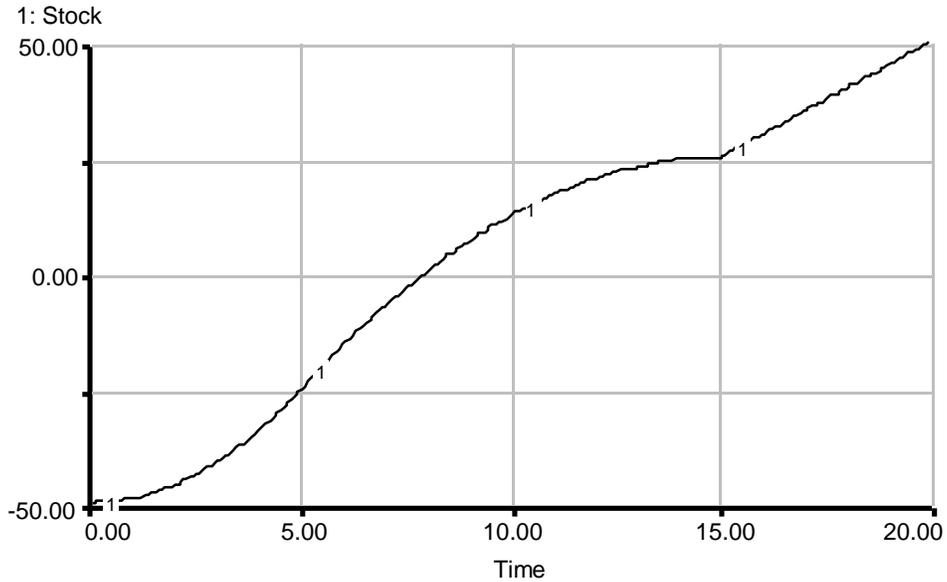
2. *Assume that the initial value of the stock is 100.*
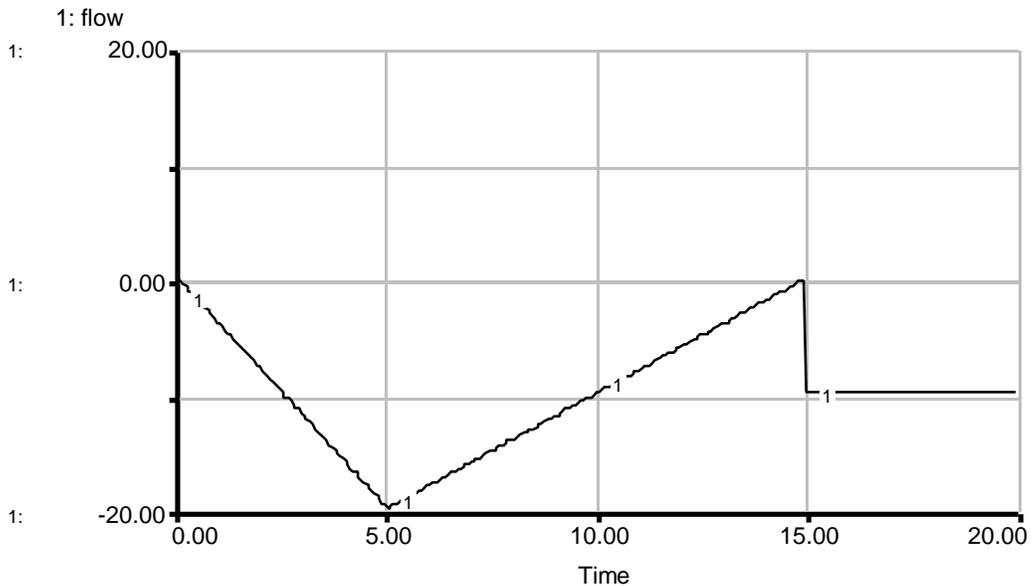
The flow again starts at 0. From time = 0 to time = 5, the flow decreases linearly with slope –4. From time = 5 to time = 15, the flow remains negative but increases linearly with slope +2. At time = 15, the flow steps down to –10 and remains constant at –10 until time = 20.

The initial value of the stock is 100, and the flow starts at 0.

From time = 0 to time = 5, the flow decreases linearly with slope –4. The change in the value of the stock, which equals the area between the graph of the flow and the zero flow line, is 5 * (–20) / 2 = –50 units of stock. The value of the stock thus decreases by 50 units, to 50 units. Because the absolute value of the flow (also called its magnitude, or its distance from zero) increases during this time segment, the absolute value of the slope of the stock graph also increases. Hence, the stock decreases more and more rapidly—the stock exhibits "decreasing" parabolic behavior.

From time = 5 to time =15, the flow remains negative but increases linearly with slope +2. The change in the value of the stock, which equals the area between the flow graph and the zero flow line, is (15 – 5) * (–20) / 2 = –100 units of stock. The value of the stock therefore decreases by 100 units, from 50 to –50 units. Because the absolute value of the flow decreases during this time segment, the absolute value of the slope of the stock graph also decreases. Hence, the stock decreases less and less rapidly—the stock exhibits parabolic behavior.

At time = 15, the flow steps down to –10 and remains constant at –10 until time = 20. The change in the value of the stock, which equals the area between the flow graph and the zero flow line, is (20 – 15) * (–10) = –50 units of stock. The value of the stock thus decreases by 50 units, to –100 units. Because the flow is constant (its value does not change) during this time segment, the slope of the stock also remains constant, and the stock decreases linearly.

Notice that the flow in part 2 is equal to –2 times the flow in part 1, and the initial value of the stock in part 2 is also equal to –2 times the initial value of the stock in part 1. The behavior of the stock in part 2 could thus simply be obtained by "reversing" the behavior of the stock in part 1 and "stretching" the graph by a factor of 2.

The stock behavior corresponding to the above description is shown below:

1: Stock
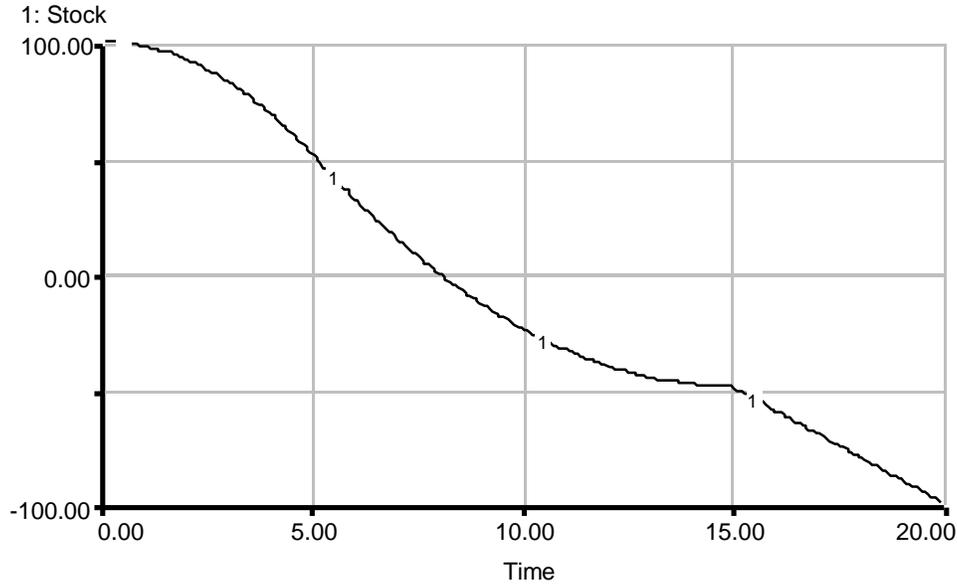
B. Although graphical integration may just seem as a useless exercise in arithmetic, an intuitive understanding of the process is often useful when trying to understand the behavior of a system. Using the insights from the two Graphical Integration Exercises papers that you have read so far, fill in the following table by trying to generalize what type of stock behavior results from various types of flow behaviors.

The table should be filled in as follows. For negative flows, it is important to define what we mean by "increasing" and "decreasing"—is it the actual value of the flow that increases or decreases, or is it the absolute value (the magnitude, or distance from zero)? In the table below, a negative flow that is "increasing" ("decreasing") is defined as a negative flow whose absolute value increases (decreases). Thus, for example, a flow that goes changes from –2 to –4 is negative and increasing. Note that for positive flows, the actual value and the absolute value are equal.

Sign of Flow	Value of Flow	Stock Behavior
zero flow	zero	not changing
positive	constant	increasing linearly
	increasing	increasing more and more rapidly
	decreasing	increasing less and less rapidly
negative	constant	decreasing linearly
	increasing	decreasing more and more rapidly
	decreasing	decreasing less and less rapidly

Several of you stated in your responses that when flows are increasing or decreasing, the stocks undergo "parabolic" or "exponential" increasing/decreasing growth/decay. From

the information given, however, it is impossible to distinguish whether the behavior of the stock will be parabolic or exponential. If the flow is a function linearly increasing in time, then the flow will be changing more and more rapidly in a *parabolic* manner. If the flow is part of a feedback loop, then as the flow increases, the stock will change more and more rapidly in an *exponential* manner. Whether the behavior is parabolic or exponential depends on whether the flow is driven exogenously or endogenously via a feedback loop. Furthermore, the flow may be neither exponential nor linear. In such a case, the stock behavior may be parabolic-like, but not parabolic. Keep in mind that exponential and parabolic behaviors have precise mathematical formulas. Hence the terms should not be used to describe patters that merely look exponential or parabolic.

How does the sign of the flow determine the stock behavior? How about the value of the flow? What is the role of the initial value of the stock?

The sign of the flow determines whether the stock is increasing (if the flow is positive) or decreasing (if the flow is negative).

The value of the flow determines how fast the stock changes. If the absolute value of the flow increases, the stock changes more and more rapidly. If the absolute value of the flow decreases, the stock changes less and less rapidly. Note that the above statements are true for any kind of flow, not just linear flows.

In simple systems such as the ones studied in this exercise, the initial value of the stock has no role in determining the behavior of the stock. The initial value is, however, essential in determining the value of the stock at any time.

3. Beginner Modeling Exercises Section 2: Mental Simulation of Positive Feedback

A. As required in the Exploration exercise #1, give three simple examples of positive feedback systems (please try to think of systems that have not been mentioned in the readings so far). For each example, provide a short verbal description (one or two sentences) of the positive feedback loop.

Weeds in your yard: The more weeds there are the more they propagate, and then there are more weeds to propagate.

My husband asked me years ago how many daffodils I wanted in the front yard—I replied, "A host." He initially planted 25 bulbs, but every other year he dug them up and found that on average each bulb had split into three viable bulbs. In a very few years, I had my host of golden daffodils.

The importance of the English language as the international language: the more people who learn English, the more it is used for international communications by scientists,

businessmen, etc., the more important English becomes, and the more people interested in learning English.

Teenage fads spread through positive feedback. When one teenager decides to try something new (like nose piercing), his friends may follow suit, then their friends do so. As the system loops and loops—the craze sweeps the country.

Children in a school with chicken pox: The stock of children with the disease increases as more and more contract chicken pox. The larger the population of children with the disease, the more children they will infect, the larger the chicken pox population.

Corruption breeds corruption: If one government official engages in corrupt practices then others will not be afraid to do the same. The more corrupt examples there are, the more people engage in the practice. The stock is the number of corrupt officials, the growth factor would be "corruption pollutant factor," which is the effect that each corrupt official has on causing others to become corrupt. The flow is the number of newly corrupt officials added.

B. Pick one of the systems you described in part A. and build a model of the system using Vensim PLE. Choose the initial value of the stock and the value of the growth fraction, formulate the equations for the model, and simulate the model. What type of behavior does the model generate? In your assignment solutions document, please include the model diagram, documented equations, and a graph of the model behavior.

Weeds in a garden:

Model diagram:
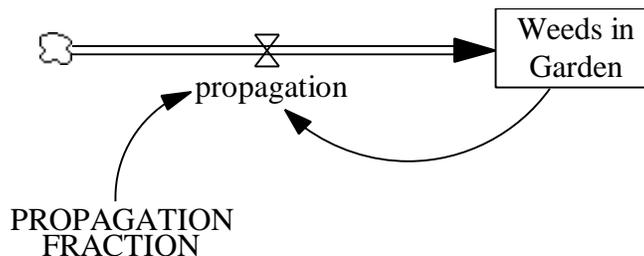
Model equations:

propagation = Weeds in Garden * PROPAGATION FRACTION
      Units: weed/week
      The number of weeds growing each week is equal to the number of weeds in the garden multiplied by the propagation fraction.

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PROPAGATION FRACTION = 0.1
      Units: 1/week
      The fraction of a weed that each existing weed produces per week.

Weeds in Garden = INTEG (propagation, 10)
      Units: weed
      The number of weeds in the garden.

Model behavior:

As expected, the model generates exponential growth of the number of weeds in a garden:
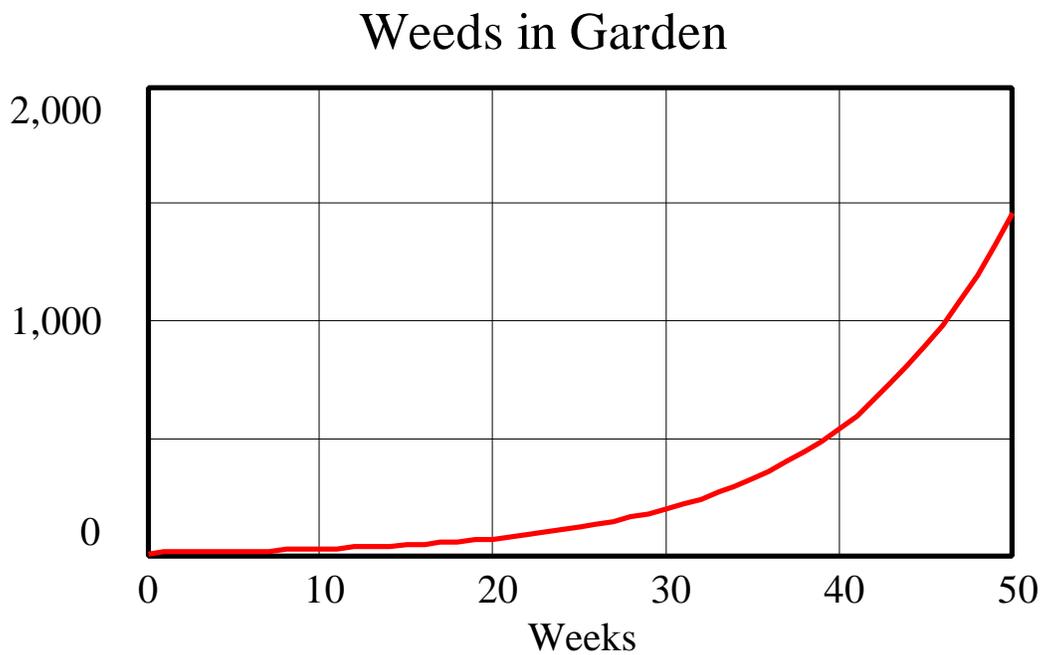
Weeds in Garden

Weeds in Garden : weeds ————————— weed

Corruption of officials:

Model diagram:

CORRUPTION
POLLUTANT FACTOR
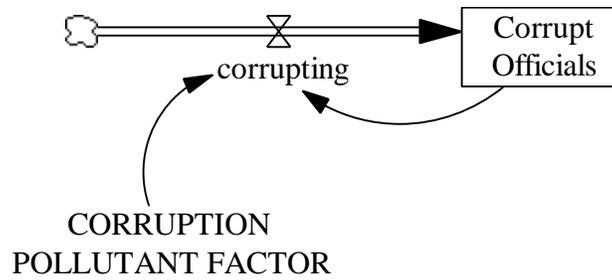
Model equations:

Corrupt Officials = INTEG (corrupting, 5)
Units: official
The number of corrupt officials.

corrupting = Corrupt Officials / CORRUPTION TIME
Units: official/Month
The number of officials who become corrupt each month.

CORRUPTION TIME = 8
Units: Month
The time it takes an already corrupt official to corrupt one of his colleagues.

Model behavior:

The model generates exponential growth of the number of corrupt officials:
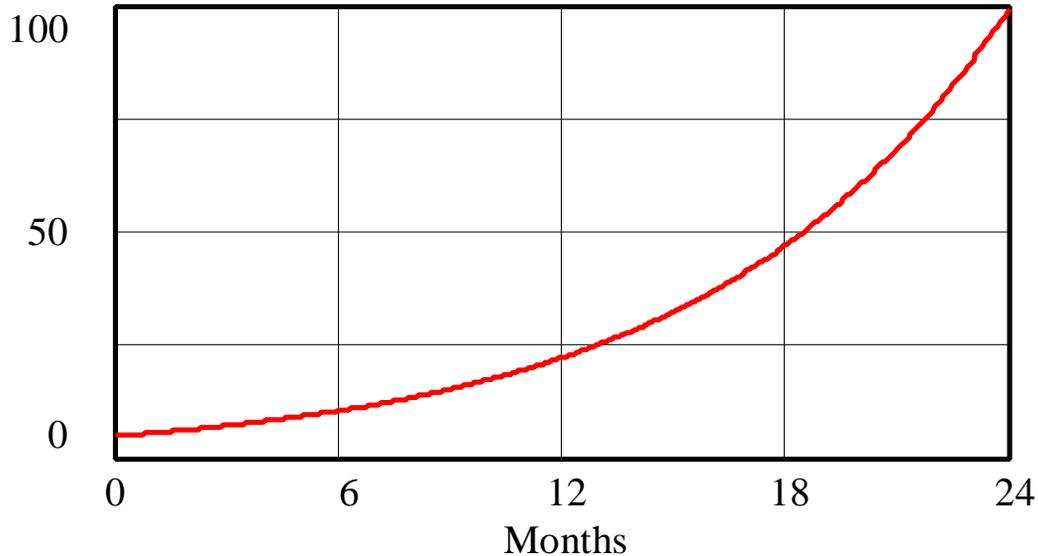
Corrupt Officials

Corrupt Officials : corrupt ——————————— official

C. What is the doubling time of the system? Explain intuitively the difference between the growth fraction and the doubling time of a system. Feel free to support your answer by deriving an equation, but we are looking for a verbal explanation here.

The doubling time of the system is 0.7/growth fraction.

It is important to realize that the doubling time and the growth fraction of a positive feedback system are not equal. The doubling time is the time it takes for the value of the stock to double. The growth fraction is the fraction of the stock that is added to the stock in each time period. Another way to think about the growth fraction is to convert it into its inverse, the time constant: time constant = 1/growth fraction. The time constant is the time it takes for each initial unit of the stock to compound into a new unit. The formula for doubling time can then be written as:

doubling time = 0.7 * time constant

All these concepts will be explained and reviewed several more times in later chapters of Road Maps.

D. Can a positive feedback system be in equilibrium? Why or why not? If yes, describe the various situations that might lead to equilibrium.

A simple positive-feedback loop can exhibit unstable equilibrium if the initial value of the stock is 0. The equilibrium is called unstable because even a small disturbance in the initial value of the stock will destroy the equilibrium and result in exponential growth. Once exponential growth takes off, the system cannot return to equilibrium.

E. As mentioned in Beginner Modeling Exercises Section 2*, growth of a positive feedback system cannot continue forever. Describe some factors that might limit the growth of the system you modeled in part B.*

Weeds in Garden: Overcrowding of the weeds will eventually reduce the propagation fraction. Exogenous inputs like application of herbicides or actual physical removal of weeds could reduce the stock. Other exogenous inputs such as rainfall amounts or temperature could effect the germination fraction.

Corruption of officials: The system of corrupt officials certainly has limits and would not grow exponentially forever. First, the total number of officials is limited so only that fixed number could become corrupt. Also, the likelihood of a corrupt official encountering a non-corrupt official will decrease as more officials are already corrupt. This would gradually decrease the growth fraction, which would gradually approach zero. It is conceivable that some officials are not corruptible.

4. Beginner Modeling Exercises Section 3: Mental Simulation of Negative Feedback

A. As required in the Exploration exercise #1, give three simple examples of negative feedback systems of the same type as the rainfall system (please try to think of systems that have not been mentioned in the readings so far). For each example, provide a short description (one or two sentences) of the negative feedback loop. Then give three examples of negative feedback systems of the same type as the solvent absorption system, and, in one or two sentences, describe the negative feedback loop for each of them. What are the differences and similarities between the two types of negative feedback systems?

When I start eating and am very hungry, I tend to eat quickly and larger quantities. As my stomach fills up, the size and frequency of my bites both diminish. Once there is "zero" room remaining in my stomach, I have no desire to continue eating.

A friend of mine is studying for the GMAT exam. Forty areas of math review require her attention. In the early stages of her preparation, she focused on the areas she knew best, reviewing material rapidly while covering lots of ground. As the review process moves along, she finds herself working on smaller and smaller components of the material. Once she's covered forty areas, the review process will end.

Using a dustpan can function as a negative feedback system. On my first pass using the dustpan, I get around 80% of the dust on the floor. On each subsequent pass, I tend to get 80% of what remains from the prior pass.

A cake in the oven behaves as the solvent absorption system. A cake has to rise to a certain height. At the beginning, it rises very quickly, but the last stretch takes a long time to complete.

When you are trying to prepare a list of items (let's say, find six examples of negative feedback systems) the first ones always come to mind very quickly, and the last ones take a very, very long time. As you complete the list, the thinking period becomes longer.

Filling your gas tank: When you first start filling the gas tank on your car you will hold the nozzle wide open. As the tank approaches full, you start to back off on the handle to slow the flow. As you approach the full mark, you are slowing to almost no flow.

The two types of systems are essentially the same. Both exhibit "goal-seeking" behavior characteristic of negative-feedback loops. In the rainfall system, the implicit goal of the "dry surface area" stock is zero, while in the solvent absorption system, the goal is modeled explicitly as the "max height." To determine whether a negative feedback loop will generate growth or decay, just compare the initial value of the stock and the goal value. If the initial value is higher than the goal, the stock will decrease towards the goal. If the initial value is lower than the goal, the stock will increase towards the goal. The goal-seeking behavior is asymptotic—each stock approaches its goal value, fast at first and then more and more slowly as time passes.

B. For each of the two types of negative feedback systems, do the following: pick one of the systems you described in part A. and build a model of the system using Vensim PLE. Choose the initial value of the stock and the values of any other parameters in the model, formulate the equations for the model, and simulate the model. What type of behavior does the model generate? Compare the behaviors generated by the two models. In your assignment solutions document, please include the model diagram, documented equations, and a graph of the model behavior for both models.

Eating until stomach is full:

Model diagram:
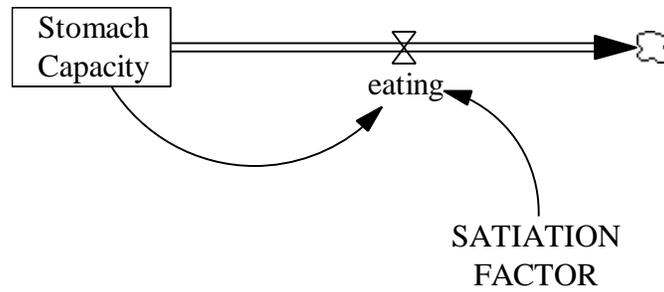
Model equations:

eating = Stomach Capacity * SATIATION FACTOR
       Units: space/Minute
       The rate at which I fill my stomach by eating more food is influenced by how much room remains in my stomach as well as the satiation factor.

SATIATION FACTOR = 0.2
       Units: 1/Minute
       The satiation factor measures the rate at which my stomach capacity diminishes because of eating.

Stomach Capacity = INTEG (-eating, 1)
       Units: space
       The remaining capacity in the stomach.

Model behavior:

The model generates asymptotic approach of "Stomach Capacity" towards zero.

Stomach Capacity
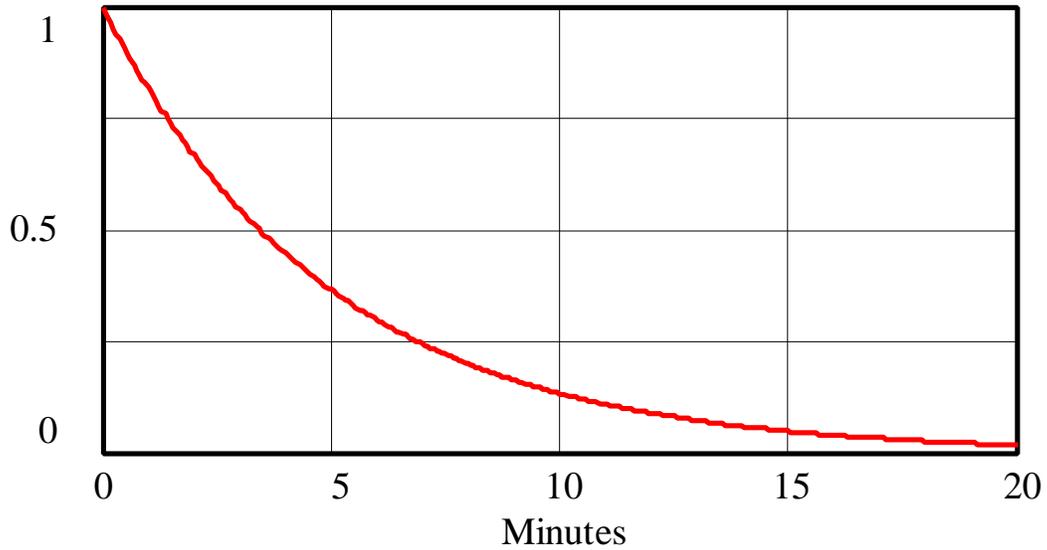
Stomach Capacity : stomach ———————————— space

Baking a cake:

Model diagram:
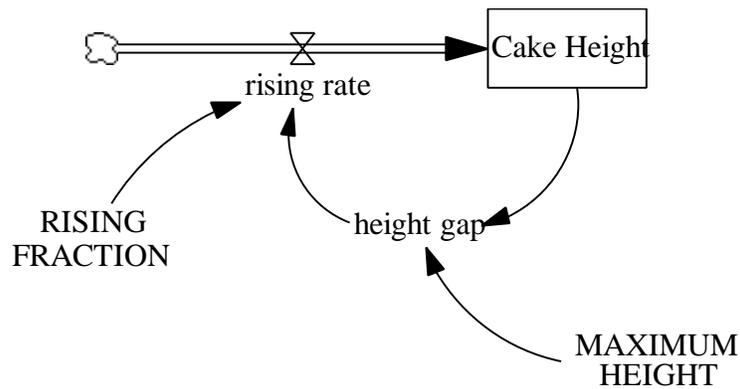
Model equations:

Cake Height = INTEG (rising rate, 8)
    Units: centimeter
    The current height of the cake.

height gap = MAXIMUM HEIGHT - Cake Height
    Units: centimeter

The difference between the cake's maximum height and its current height.

MAXIMUM HEIGHT = 22
        Units: centimeter
        The height of the cake when it is completely baked.

RISING FRACTION = 0.05
        Units: 1/Minute
        The fraction by which the cake rises every minute.

rising rate = height gap * RISING FRACTION
        Units: centimeter/Minute
        The rate at which the cake rises.

Model behavior:

As expected, the model generates asymptotic behavior, with the "Cake Height" approaching the goal of "MAXIMUM HEIGHT":
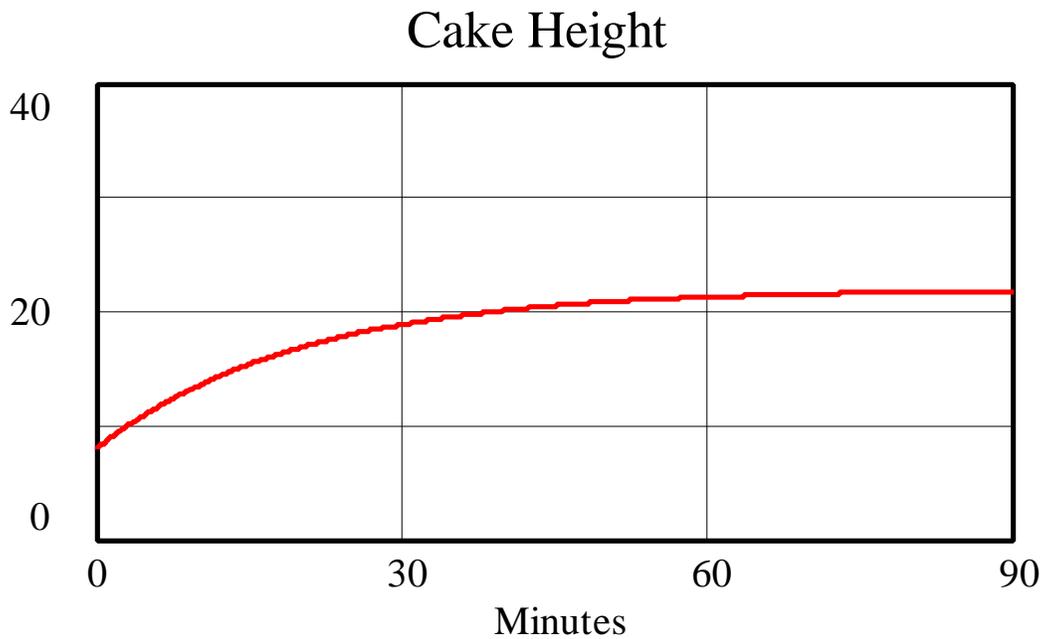
Cake Height

Cake Height : cake ————————————— centimeter

C. Calculate the half-lives for both systems. Explain intuitively the difference between the decay or growth fraction and the half-life of a system. Feel free to support your answer by deriving an equation, but we are again looking for a verbal explanation here.

The half-life for a first-order negative-feedback system equals 0.7/decay (or growth) fraction of the system.

Again, it is important to realize that the half-life and the decay or growth fraction of a negative-feedback system are not equal. The half-life is the time it takes for the stock to close one half of the gap between the value of the stock and the goal value (the goal in the rainfall-type model is 0).